



Lecture 3: Planetary Heat & Energy Transport

Planetary Systems

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Learning objectives

By the end of this lecture, you will be able to:

- Name the four energy sources of planetary interiors and rank them for Earth
- Derive the conductive cooling timescale $\tau \sim L^2/\kappa$ and use the Rayleigh number
- Explain why tidal heating makes Io the most volcanic body in the solar system

Recap: Lecture 2

- Disks live 3 to 5 Myr; dust grows to planetesimals and planets
- Giant planets: core accretion or gravitational instability
- Kepler's laws, vis-viva, resonances, tides, migration

Today: What happens *inside* planets after they form?

A grayscale image of the planet Io, showing its heavily cratered surface. A bright, elongated plume of material is visible on the left side of the planet, extending into space. The text "1. Energy sources" is overlaid in white on the right side of the planet.

1. Energy sources

What heats a planet?

1. **Accretional heating:** gravitational energy of assembly
2. **Gravitational differentiation:** core formation, about 10% more
3. **Radioactive decay:** sustained for billions of years
4. **Tidal heating:** orbital energy dissipated as friction

No internal heat means no volcanism, no magnetic field, no plate tectonics, no outgassing.

Accretional heating

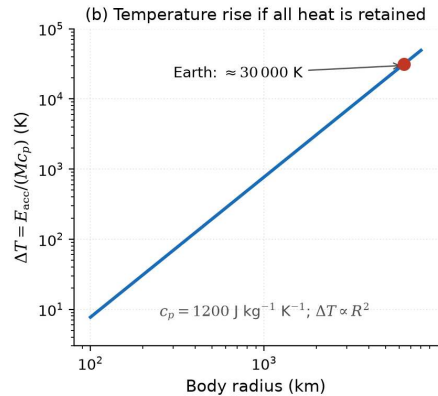
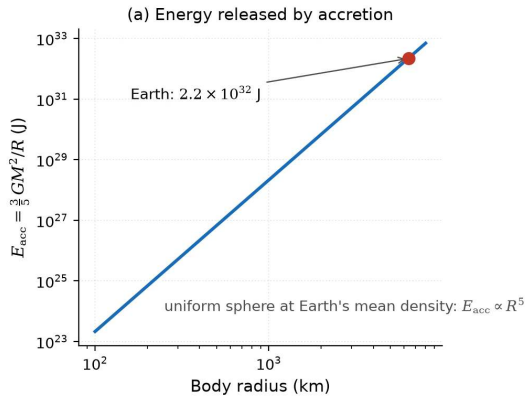
Gravitational energy of assembling a uniform sphere:

$$E_{\text{acc}} \sim \frac{3}{5} \frac{GM^2}{R} \approx 2.2 \times 10^{32} \text{ J for Earth}$$

If all of it is retained as heat:

$$\Delta T = \frac{E_{\text{acc}}}{Mc_p} \approx \frac{2.2 \times 10^{32}}{5.97 \times 10^{24} \times 1200} \approx 30,000 \text{ K}$$

Enough to melt Earth many times over: accretion produces a **magma ocean**.



Accretional energy and retained temperature rise against body size for a uniform sphere at Earth's density. Course figure.

Only 10 to 50% of E_{acc} is retained (radiation to space, thermal blanketing, depth of deposition); 10% still melts an Earth-sized planet, and magma oceans are expected from about Mars size upward.

Where do heat-producing elements live?

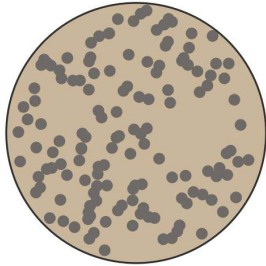
- **Lithophile** (“rock-loving”): U, Th, K stay in the silicate mantle and crust
- **Siderophile** (“iron-loving”): Fe, Ni, Co, Pt, Ir, Au go to the core

Radiogenic heat is produced in the **mantle**, not the core, and that sustains mantle convection.

Source: Goldschmidt (1937); McDonough (2003)

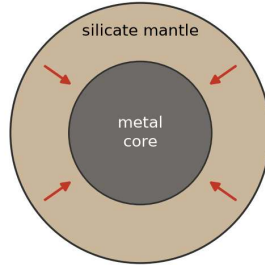
Gravitational differentiation (schematic)

(a) Homogeneous body after accretion



metal droplets dispersed in silicate rock

(b) After core formation



dense metal sinks: $E_{\text{diff}} \approx 2 \times 10^{31}$ J for Earth

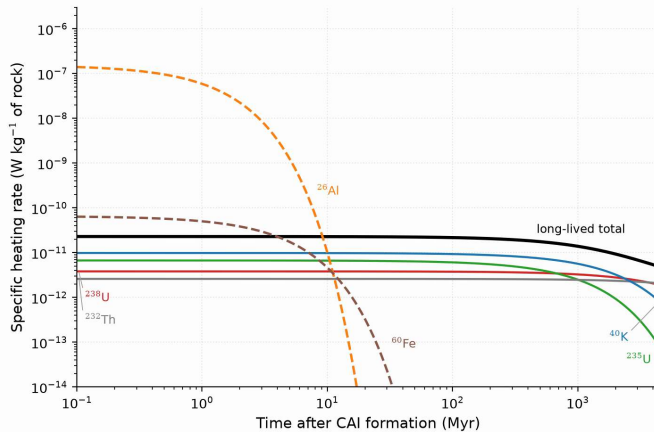
Metal droplets dispersed after accretion (a) sink to form a core (b). Course schematic.

$E_{\text{diff}} \sim \frac{3}{10} \frac{GM_{\text{core}}^2}{R_{\text{core}}} \left(\frac{R}{R_{\text{core}}} - 1 \right) \approx 2 \times 10^{31} \text{ J for Earth, about 10\% of accretion, released once during core formation.}$

Radioactive decay: the long-lived engine

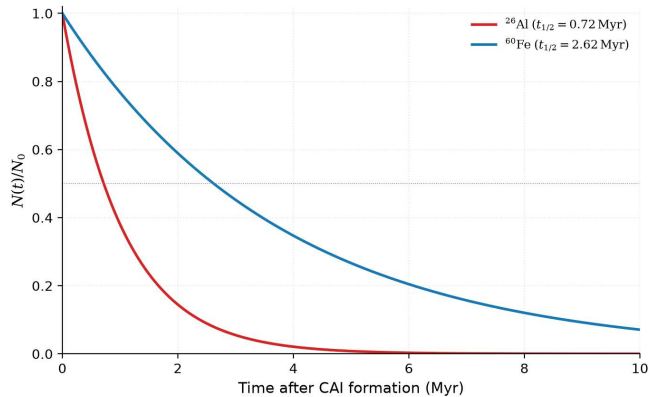
Isotope	Half-life (Gyr)	Heat (W kg^{-1})	Concentration
^{238}U	4.47	9.5×10^{-5}	20 ppb
^{235}U	0.704	5.7×10^{-4}	0.14 ppb
^{232}Th	14.0	2.6×10^{-5}	80 ppb
^{40}K	1.25	2.9×10^{-5}	28 ppb

Present-day radiogenic heat about 20 TW; 4.5 Gyr ago a factor 4.7 higher.



Heating rate of the extinct ²⁶Al and ⁶⁰Fe (dashed) and of the four long-lived isotopes (solid) over solar system history. Course figure.

Long-lived decay gives 92 TW at formation and 19 TW today; the crossover near 10 Myr hands the heating from planetesimals to planets.



Decay of ^{26}Al (half-life 0.72 Myr) and ^{60}Fe (2.6 Myr) over the first 10 Myr. Course figure.

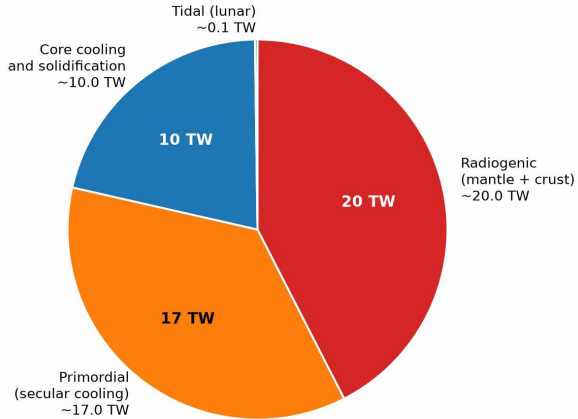
The short-lived ^{26}Al at the canonical $^{26}\text{Al}/^{27}\text{Al} \sim 5 \times 10^{-5}$ gives $10^{-7} \text{ W kg}^{-1}$: it melts planetesimals of a few km and is why small asteroids show melting and differentiation.

Earth's heat budget: 47 TW

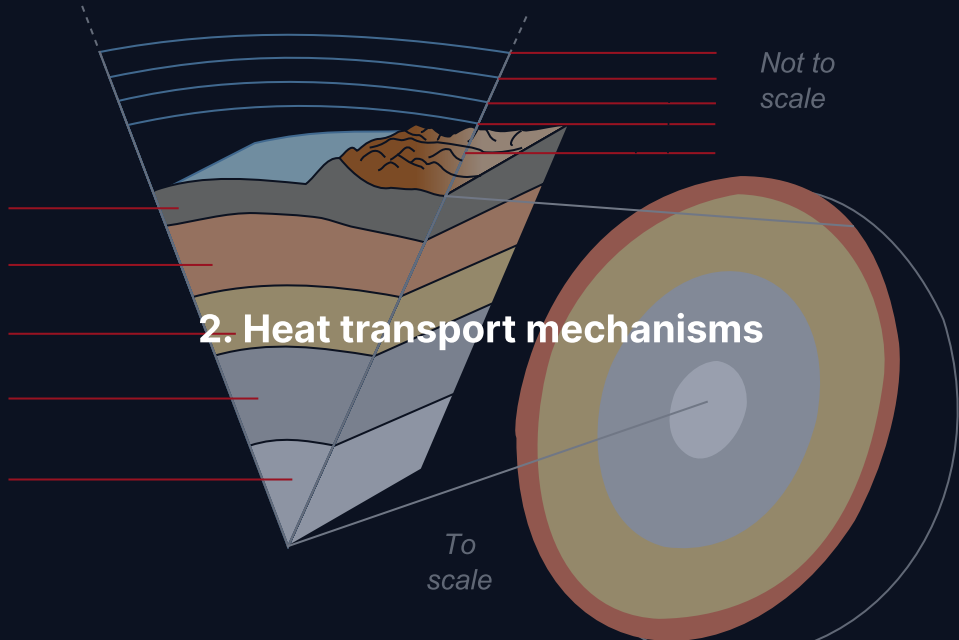
Source	Contribution
Radiogenic heating (mantle + crust)	~20 TW
Primordial (secular cooling)	~17 TW
Core cooling and solidification	~10 TW
Tidal dissipation (from the Moon)	~0.1 TW
Total	~47 TW

About 40% from ongoing radioactive decay, 60% from primordial heat:
Earth is still cooling.

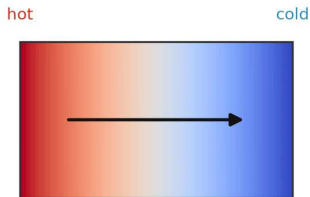
Earth's surface heat-flux budget (~ 47 TW)



Earth's 47 TW by source: radiogenic decay (red), primordial heat (orange), core cooling, and the tiny lunar tidal share. Course figure.

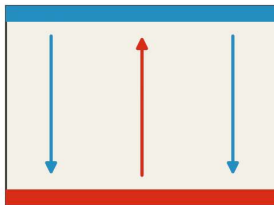


(a) Conduction



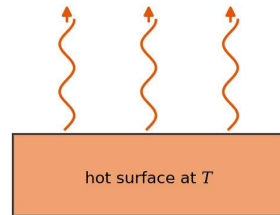
$$\vec{q} = -k\nabla T: \text{no bulk motion}$$

(b) Convection



hot fluid rises, cold fluid sinks

(c) Radiation



$$F = \epsilon\sigma T^4: \text{photons carry the energy}$$

Three ways to move heat: conduction down a temperature gradient, convection by bulk flow, radiation by photons. Course schematic.

Conduction rules lithospheres and small bodies, convection rules mantles, cores and giant-planet interiors, radiation ($F = \epsilon\sigma T^4$) rules surfaces, thin atmospheres and magma ocean surfaces; inside rock it matters only above about 2000 K.

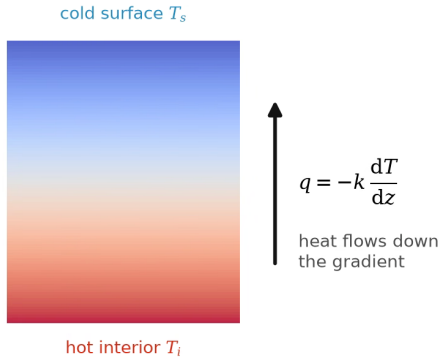
Conduction: Fourier's law and the diffusion timescale

- Heat flux $\vec{q} = -k\nabla T$
- Thermal diffusivity
 $\kappa = k/(\rho c_p) \approx 10^{-6} \text{ m}^2 \text{ s}^{-1}$ for rock
- Rock and iron have similar ρc_p
(about $3.5 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$); the
higher k of iron gives it about 20×
the κ of rock

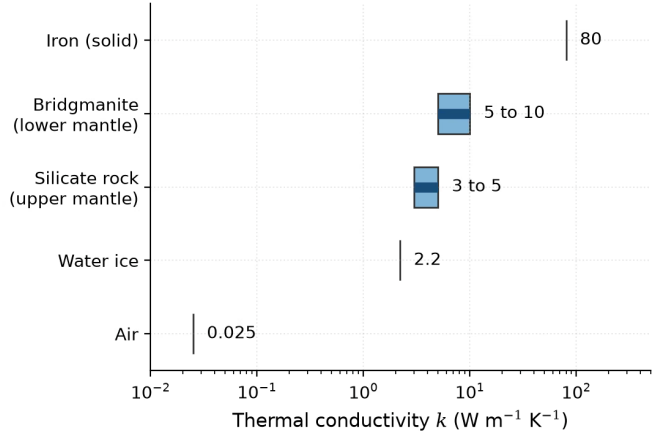
$$\tau_{\text{cond}} \sim \frac{L^2}{\kappa}$$

Material	k (W m ⁻¹ K ⁻¹)	κ (m ² s ⁻¹)
Silicate rock	3–5	$\sim 10^{-6}$
Bridgmanite	5–10	$\sim 2 \times 10^{-6}$
Water ice	2.2	1.1×10^{-6}
Iron (solid)	80	$\sim 2 \times 10^{-5}$
Air	0.025	$\sim 2 \times 10^{-5}$

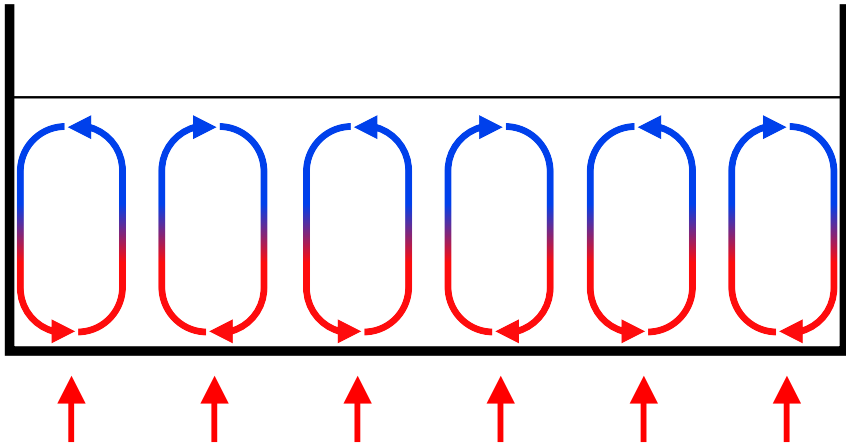
(a) Fourier's law in a slab



(b) Conductivity of planetary materials

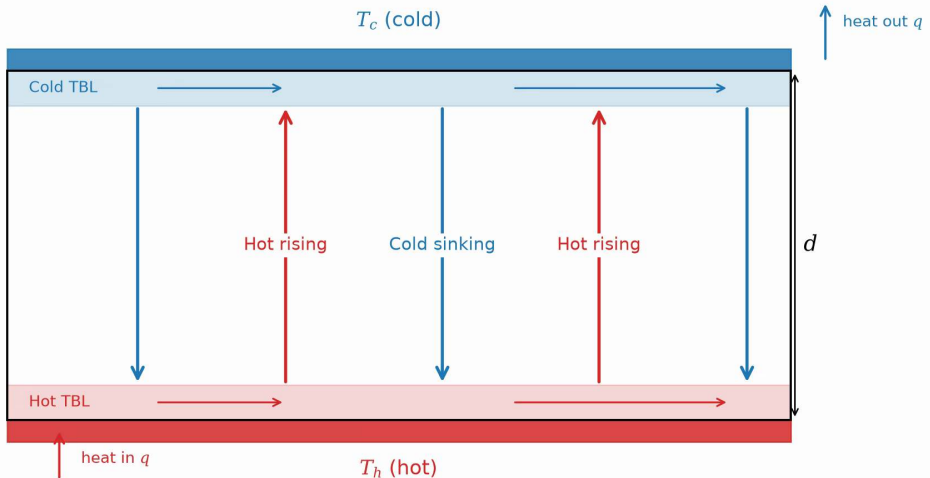


Fourier's law in a slab, and the thermal conductivity of planetary materials from air (0.025) to solid iron ($80 \text{ W m}^{-1} \text{K}^{-1}$). Course figure.

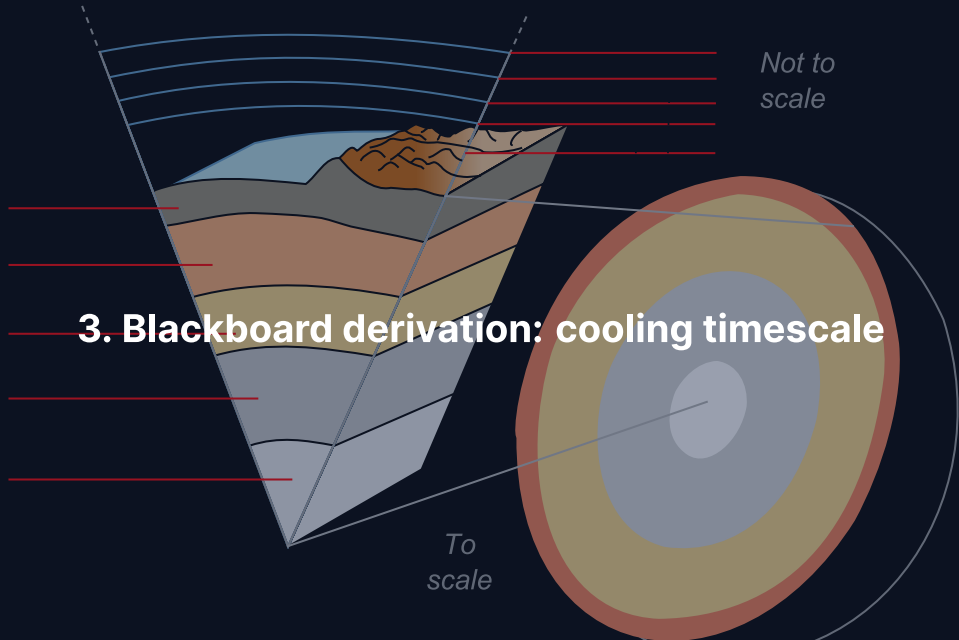


Convection cells: hot material rises from the base, cools, and sinks. Credit: Wikimedia Commons, CC BY-SA 3.0.

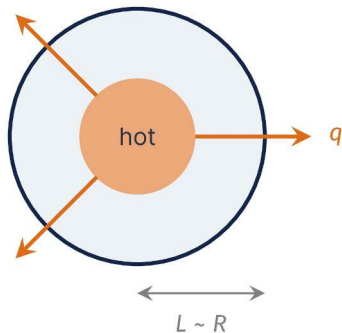
Hot rises, cold sinks: convection moves heat far faster than conduction in Earth's mantle (cm per year), the outer core, giant-planet interiors and atmospheres.



Rayleigh-Bénard convection: plumes rise from a thin basal boundary layer, cold fluid sinks from the top, lateral flow closes the cells. Course schematic.



Goal and strategy



Goal: how long does heat take to conduct out of a body of size L ?

- Write the heat equation
- Replace derivatives by scales:
 $\tau_{\text{cond}}(L, \kappa)$
- Put in rock values and compare with 4.6 Gyr

Bigger bodies have a longer path for heat: we derive *how much* longer.

Step 1: the heat diffusion equation

Fourier's law $\vec{q} = -k\nabla T$ and energy conservation give:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

For constant k , with $\kappa = k/(\rho c_p)$, in one dimension:

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T = \kappa \frac{\partial^2 T}{\partial x^2}$$

The temperature changes at a rate set by the **curvature** of the profile: flat profiles do not evolve.

Step 2: dimensional analysis

Replace derivatives with characteristic scales:

$$\frac{\partial T}{\partial t} \sim \frac{T}{\tau}, \quad \frac{\partial^2 T}{\partial x^2} \sim \frac{T}{L^2} \quad \Rightarrow \quad \frac{T}{\tau} \sim \kappa \frac{T}{L^2}$$

Temperature cancels: the timescale is intrinsic to the body.

$$\tau_{\text{cond}} \sim \frac{L^2}{\kappa}$$

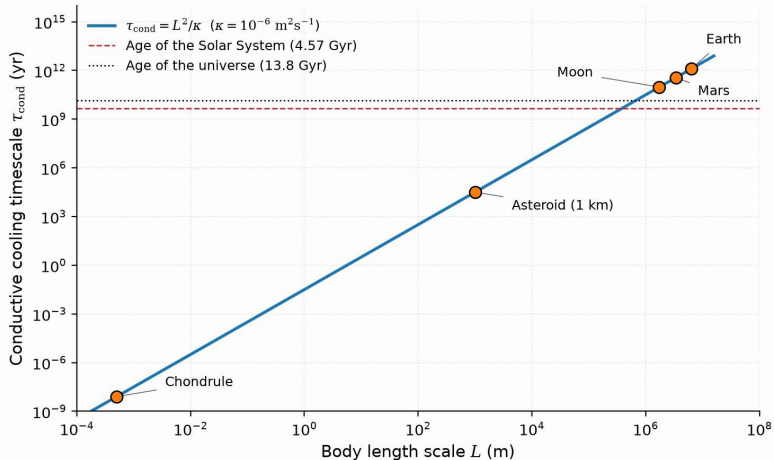
Key feature: the L^2 scaling. Doubling the size gives 4× the cooling time.

Cooling timescales across the solar system

Using $\kappa \approx 10^{-6} \text{ m}^2 \text{ s}^{-1}$ (silicate rock):

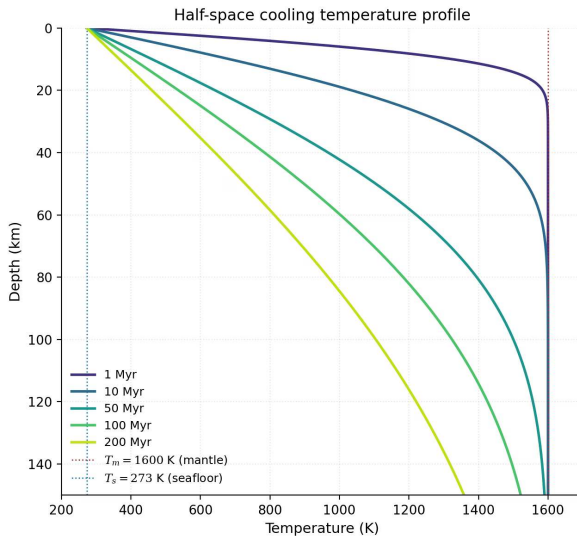
Body	Size L	$\tau = L^2 / \kappa$	Familiar units
1 m boulder	1 m	10^6 s	~12 days
100 km asteroid	10^5 m	10^{16} s	~300 Myr
Moon	$1.7 \times 10^6 \text{ m}$	$3 \times 10^{18} \text{ s}$	~100 Gyr
Earth	$6.4 \times 10^6 \text{ m}$	$4 \times 10^{19} \text{ s}$	~1300 Gyr

Earth's conductive cooling time $\gg$ age of the universe.
Yet Earth loses 47 TW $\Rightarrow$ there **must** be convection.



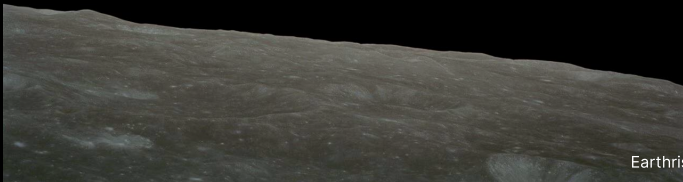
Conductive cooling time $\tau = L^2/\kappa$ against body size, slope 2 on log axes. Course figure.

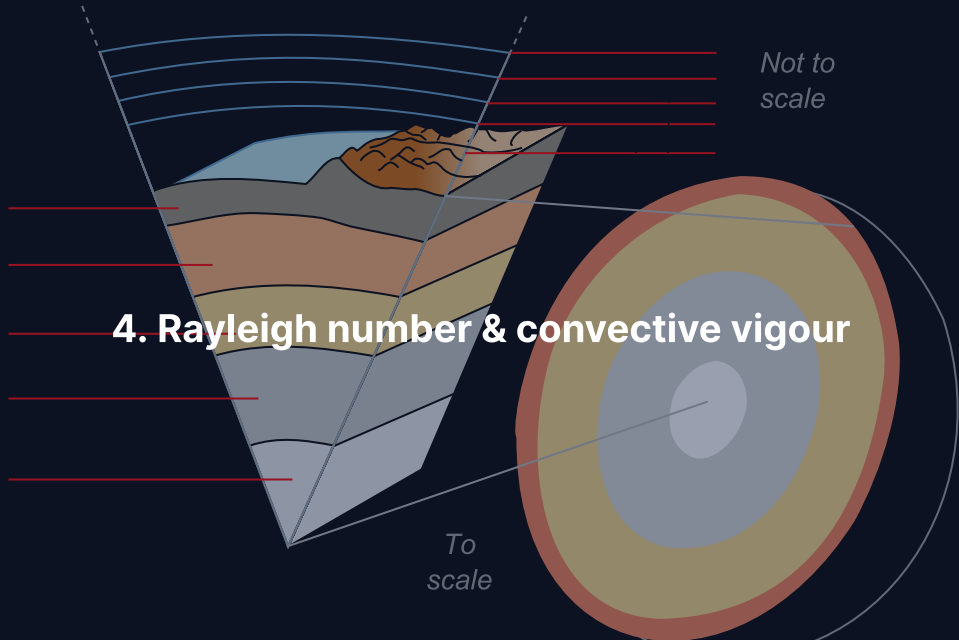
A chondrule cools in under a second, a 1 km asteroid in 30 000 yr; any body larger than a few hundred km must convect, or stay hot.



Half-space cooling for ages 1 to 200 Myr: the lithosphere thickens as $\sqrt{kt}$ and the heat flux falls as $t^{-1/2}$. Course figure, after Turcotte & Schubert (2002).

Break





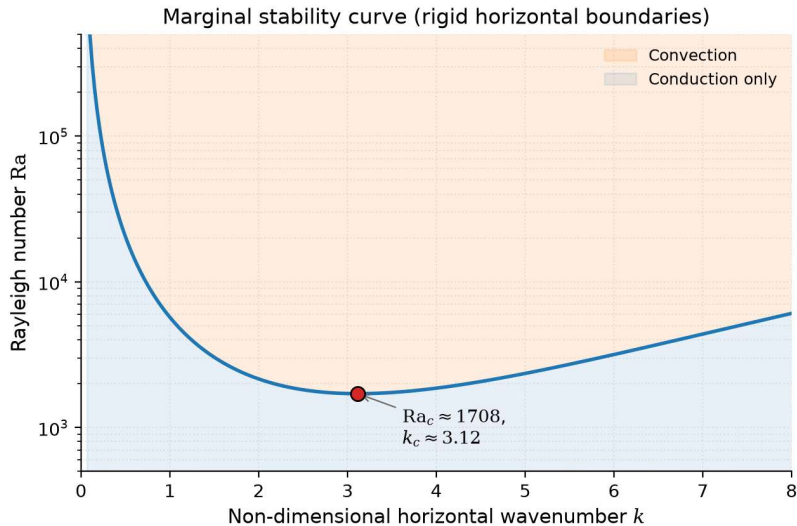
The Rayleigh number

Does a fluid layer convect or merely conduct?

$$\text{Ra} = \frac{\alpha \rho g \Delta T d^3}{\kappa \eta} \quad \frac{\text{buoyancy}}{\text{dissipation}}$$

α thermal expansion, ρ density, g gravity, ΔT temperature contrast, d layer depth; κ thermal diffusivity, η dynamic viscosity.

Convection starts above $\text{Ra}_c \approx 1708$ (rigid boundaries; 658 for free-slip).



Marginal stability: conduction below the curve, convection above; the minimum $Ra_c \approx 1708$ sits at $k_c \approx 3.12$. Course figure, after Turcotte & Schubert (2002).

Earth's mantle: vigorous convection, and how efficient

$$Ra_{\oplus} \sim \frac{2 \times 10^{-5} \times 4000 \times 10 \times 2500 \times (3 \times 10^6)^3}{10^{-6} \times 10^{21}} \sim 10^7 - 10^8 \gg Ra_c$$

- Nusselt number $Nu = q_{\text{total}}/q_{\text{cond}}$: efficiency, $Nu = 1$ is pure conduction
- Boundary-layer theory: $Nu \propto Ra^{1/3}$, a factor 10^3 in Ra gives 10 in Nu

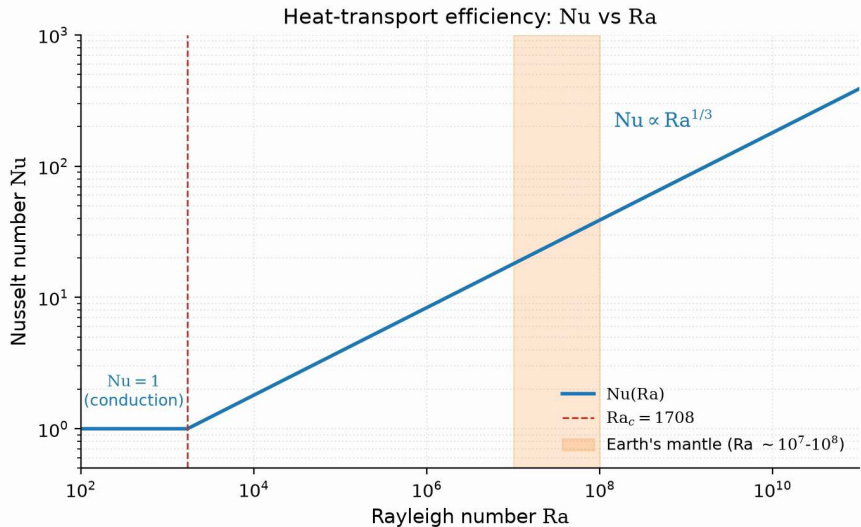
For Earth, $Nu = (Ra/Ra_c)^{1/3} \approx 18\text{--}39$: convection moves heat tens of times faster than conduction.

Source: Schubert et al. (2001), *Mantle Convection in the Earth and Planets*

Rayleigh number across the solar system

Body	Ra	Regime
Earth's mantle	10^7 – 10^8	Vigorous convection
Venus' mantle	10^7 – 10^8	Vigorous (stagnant lid)
Mars' mantle	10^6 – 10^7	Moderate convection
Moon's mantle	10^4 – 10^5	Weak; cooling by conduction
Mercury's mantle	$\sim 10^4$	Marginal; largely cooled
Magma ocean	$\geq 10^{25}$	Turbulent convection
Earth's outer core	$\geq 10^{25}$	Turbulent (dynamo)

Ra scales as d^3/η : small bodies and stiff interiors drop below Ra_c quickly.



$Nu = 1$ below Ra_c , then $Nu \propto Ra^{1/3}$; Earth's mantle (shaded) moves heat 18 to 39 times faster than conduction. Course figure.

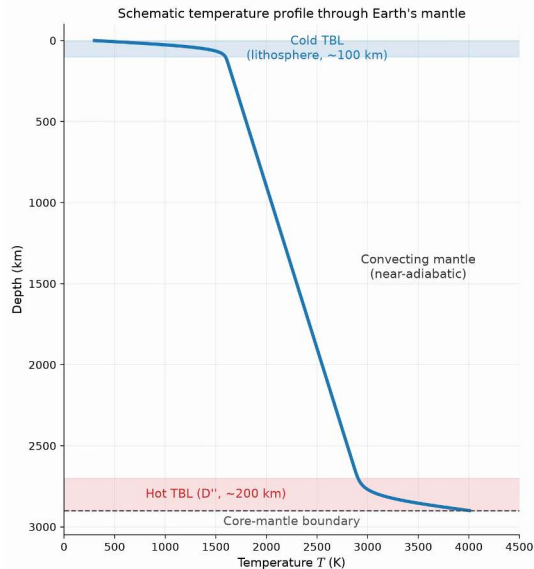
The adiabatic gradient

In the well-mixed convecting interior, temperature follows an **adiabat**:

$$\left(\frac{dT}{dr}\right)_{\text{adiabat}} = -\frac{\alpha g T}{c_p}$$

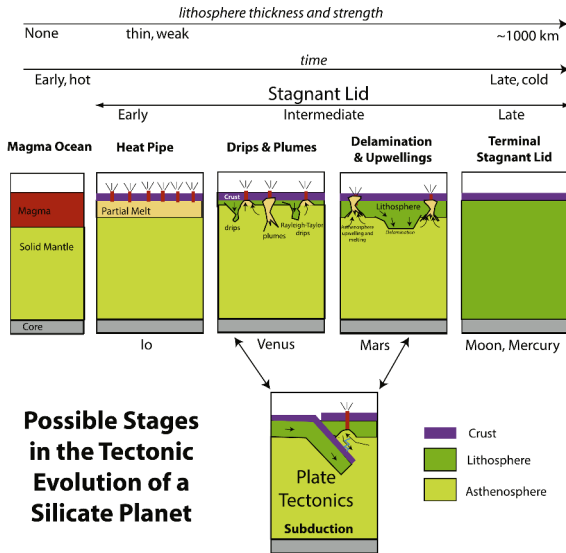
- Earth's upper mantle: about 0.3 to 0.5 K km⁻¹
- Conductive gradient in the lithosphere: about 25 K km⁻¹

A gradient of only about 0.5 K km⁻¹ across the convecting mantle: efficient mixing keeps the interior close to an adiabat.



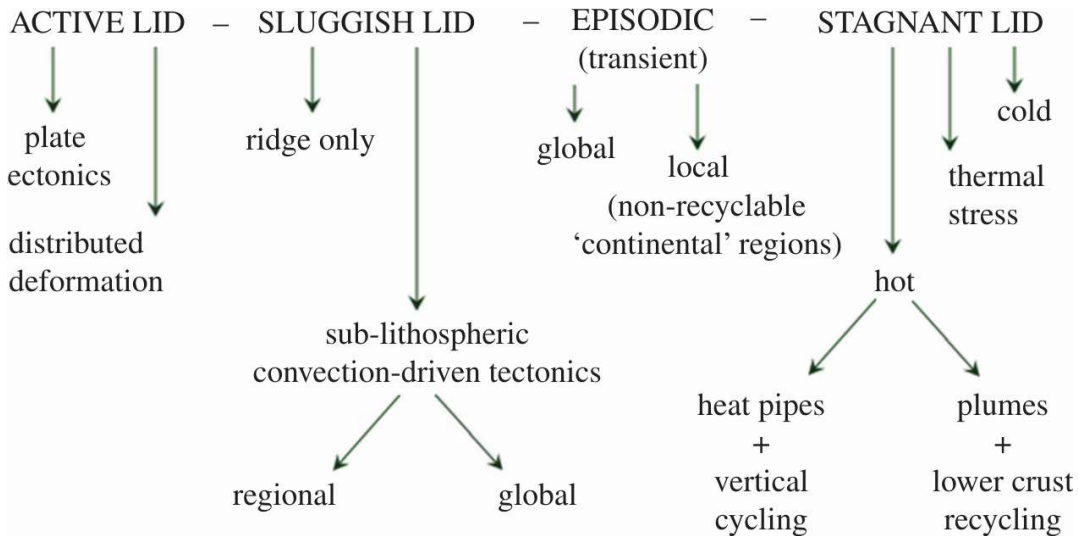
Earth's mantle temperature:
conductive lithosphere, adiabatic
interior at about 0.5 K km^{-1} , and the
hot D'' layer above the core.
Schematic after Schubert et al. (2001).

Two thin conductive boundary
layers, the lithosphere (100 to
200 km) and D'' (about 200 km),
hold the steep parts of the
profile; between them the
mantle follows the adiabat.

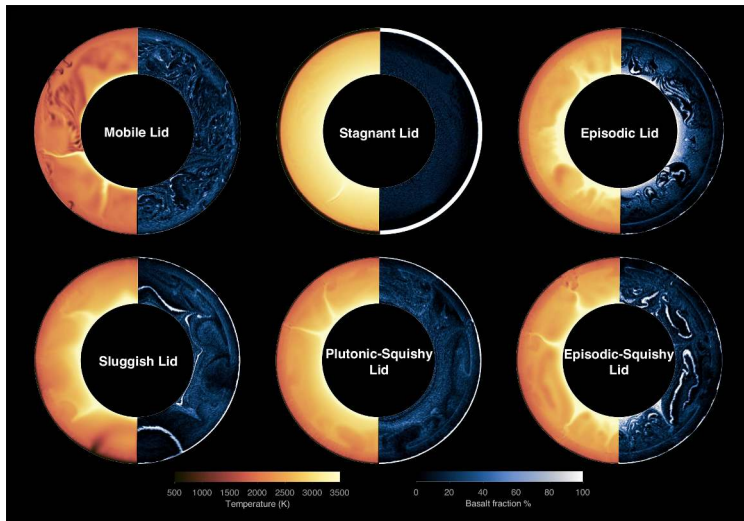


Stages of a planet's tectonic evolution. Reproduced from Stern et al. (2018), Fig. 3.

Mobile lid: the cold boundary layer breaks into plates that subduct (Earth only). **Stagnant lid:** one immobile shell cooled by conduction and volcanism (Venus, Mars, Mercury, Moon).



The taxonomy of tectonic modes: active lids, episodic modes, and cold or hot stagnant lids as end-members of one spectrum. Reproduced from Lenardic (2018), Fig. 2.



Six lid regimes in mantle convection simulations (temperature left, basalt fraction right): mobile, stagnant, episodic, sluggish, plutonic-squishy, episodic-squishy. Reproduced from Lyu et al. (2025), Fig. 1, CC BY 4.0.

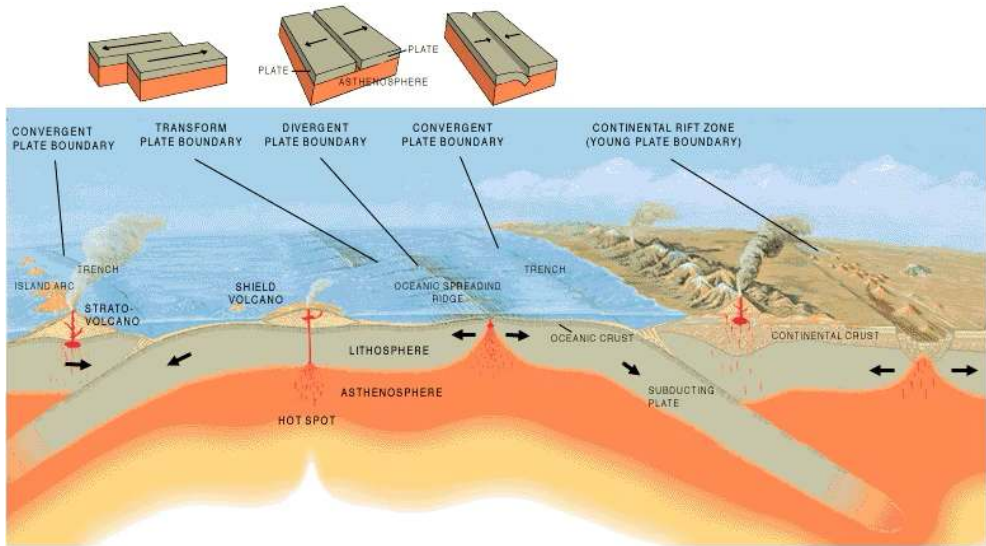
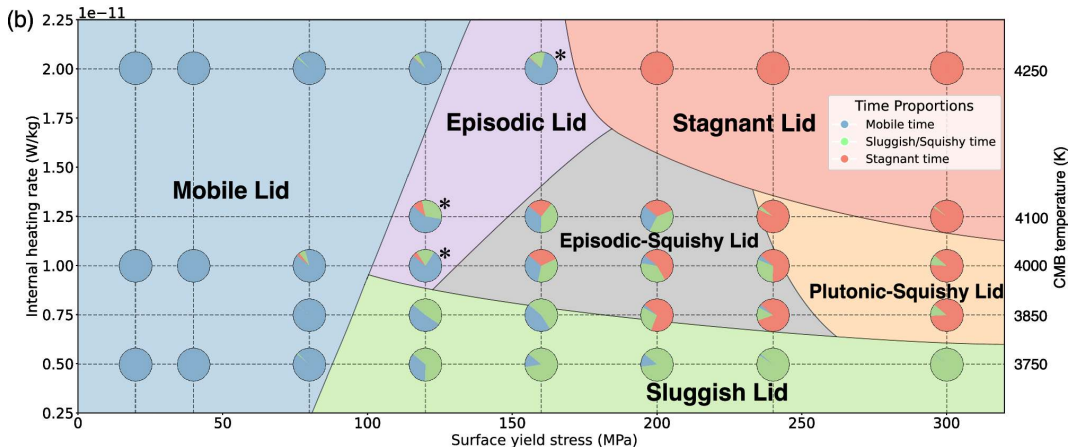
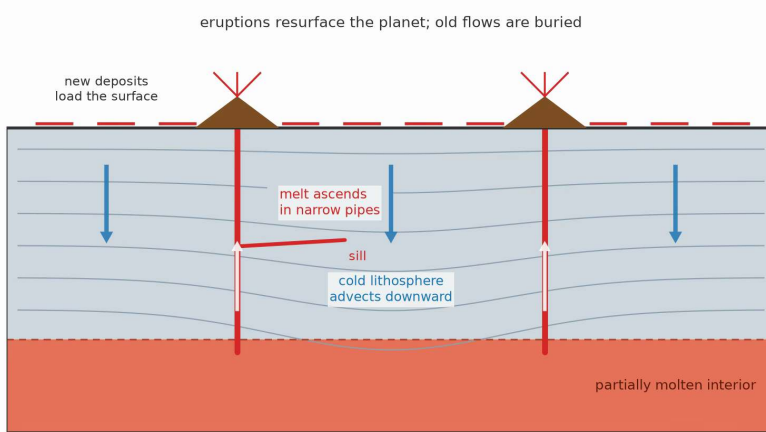


Plate-tectonic settings in one cross-section: ridges, subduction at convergent margins, and a plume rising independently of the plates. Credit: USGS, public domain.



Simulated tectonic regime against lithosphere strength and internal heating. Reproduced from Lyu et al. (2025), Fig. 5b, CC BY 4.0.

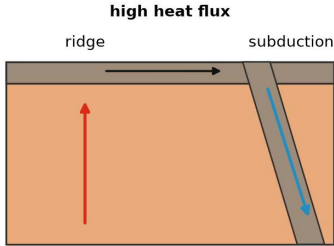
Why only Earth: water, planet size, thermal state and prior history together; plate tectonics, liquid water and long-term climate stability are linked.



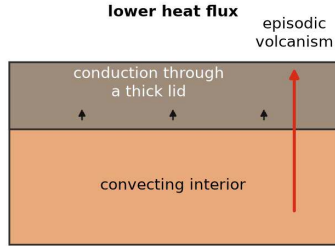
Heat-pipe volcanism: magma rises through conduits, erupts, and buries the older crust.
Course schematic, after Moore & Webb (2013).

A very hot mantle cools by erupting and burying lava: Io today (resurfaced every $\sim 10^6$ yr), and possibly the Hadean Earth.

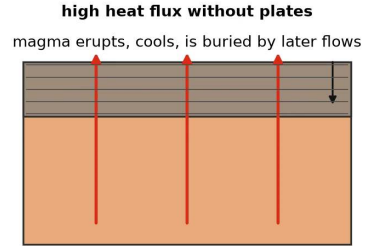
(a) Mobile lid (plate tectonics)



(b) Stagnant lid



(c) Heat pipe (Io, early Earth)



Heat loss under three regimes: a mobile lid at ridges and subduction zones, a stagnant lid by conduction and volcanism, a heat pipe by erupted magma. Course schematic.

EXPLANATION

-  Divergent plate boundaries—Where new crust is generated as the plates pull away from each other.
-  Convergent plate boundaries—Where crust is consumed in the Earth's interior as one plate dives under another.
-  Transform plate boundaries—Where crust is neither produced nor destroyed as plates slide horizontally past each other.
-  Plate boundary zones—Broad belts in which deformation is diffuse and boundaries are not well defined.
-  Selected prominent hotspots

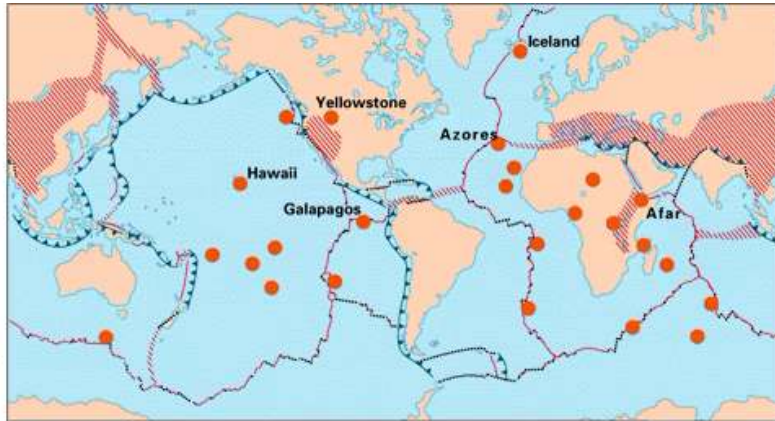
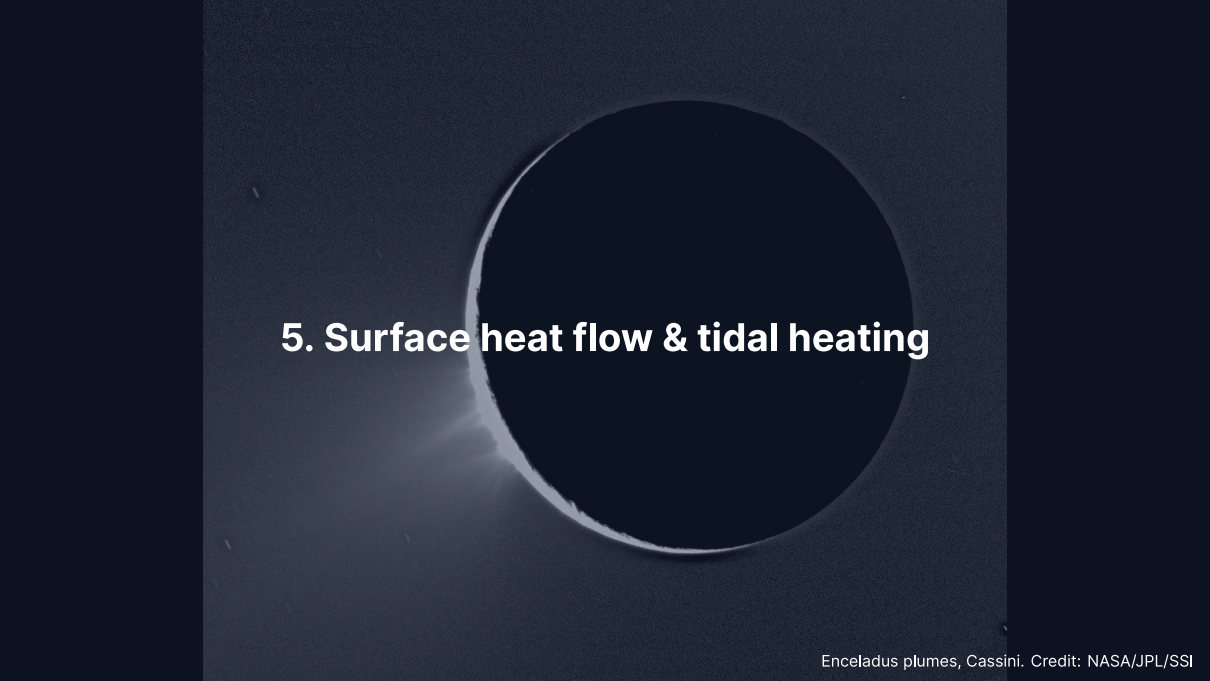


Plate boundaries and hotspots: on ridges (Iceland, Azores) and inside plates (Hawaii, Yellowstone). Credit: USGS, public domain.

Plumes about 100 km wide detach from D'', live 10^7 to 10^8 yr, and feed intraplate volcanism: heat from the core-mantle boundary bypasses the plates.



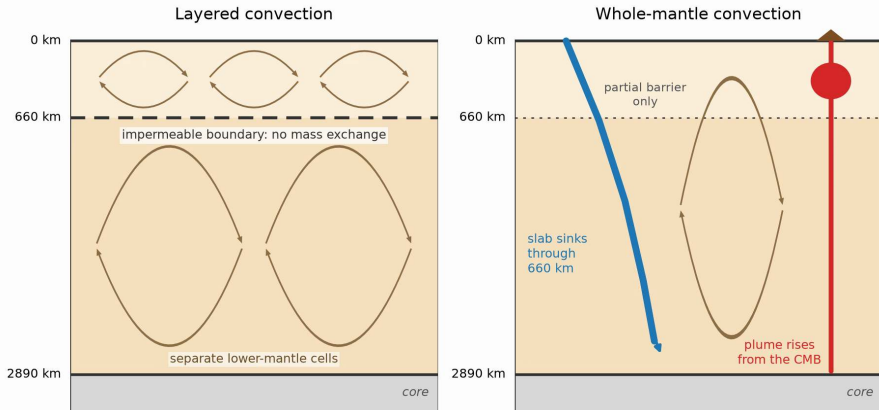
5. Surface heat flow & tidal heating

Core cooling and the dynamo

The mantle is the thermostat of the core.

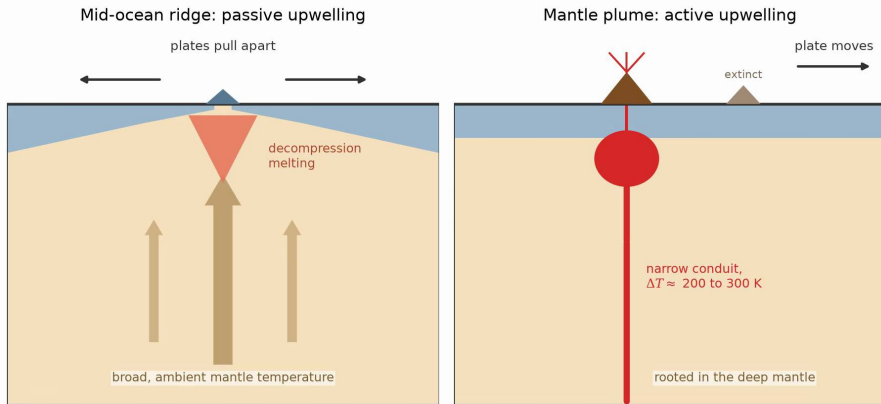
- A convecting mantle extracts core heat and lets the core convect
- Core cooling crystallises the inner core: latent heat and light elements
- This compositional buoyancy drives today's geodynamo (Lecture 4)

Earth's field exists because mantle convection cools the core; stagnant-lid Venus likely lacks this.



Whole-mantle against layered convection. Course schematic; tomographic evidence from Grand (2002) and Ritsema et al. (2011).

Seismic tomography shows the Farallon slab sinking to the core-mantle boundary: **whole-mantle convection**; the 660 km boundary slows slabs but does not stop them.



Two modes of upwelling: passive at mid-ocean ridges, active in mantle plumes. Course schematic; after Courtillot & Renne (2003).

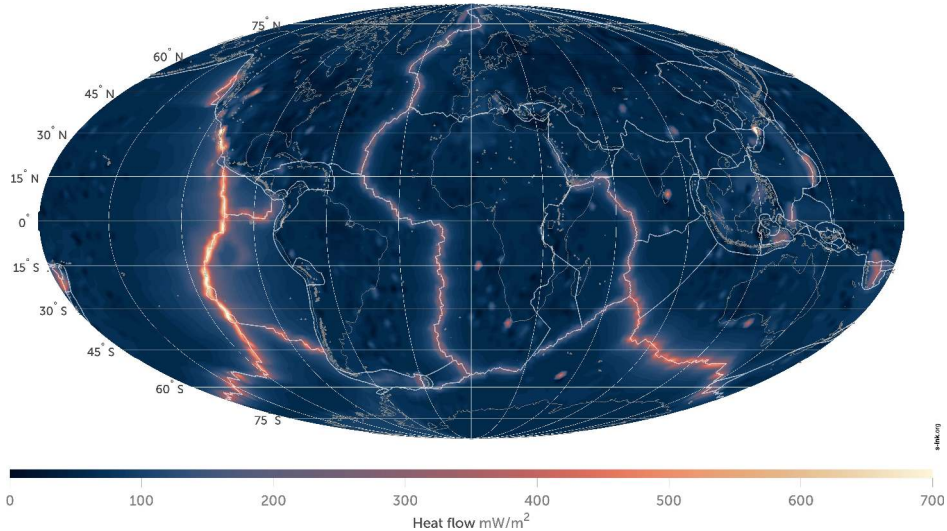
Ridges upwell at ambient temperature; plumes are 200 to 300 K hotter and ^3He -rich, and plume heads build Large Igneous Provinces such as the Deccan Traps.

Surface heat flow

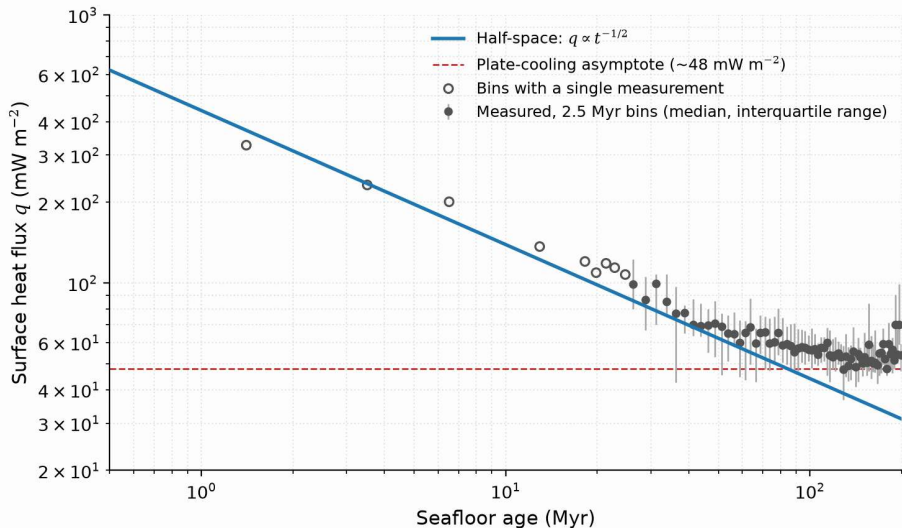
Body	Heat flux (mW m^{-2})	Total (TW)	Source
Earth	~90	~47	Radiogenic + primordial
Moon	10–15	~0.3	Primordial (cooled)
Mars	15–25	~2–4	Radiogenic + primordial
Io	2000–2500	~105	Tidal dissipation

Source: Davies & Davies (2010); Davies et al. (2024); InSight + gravity/topography estimates

Io emits 105 ± 12 TW, twice Earth's 47 TW, from 1.5% of Earth's mass; Earth loses 70% of its heat through the oceans.



Surface heat flow from about 70 000 measurements: hottest at mid-ocean ridges, coldest on old continental shields. Credit: Fabio Cramerì (Wikimedia, CC BY-SA 4.0); data from Lucazeau (2019).



Seafloor heat flux against crustal age: the half-space $t^{-1/2}$ prediction (blue) against binned measurements (grey). Data: Richards et al. (2018); asymptote after Stein & Stein (1992).

Secular cooling of planets

- Stored heat $\sim E_{\text{acc}} + E_{\text{diff}} + \int L_{\text{rad}}(t) dt$; lost heat $\sim \int L_{\text{surface}}(t) dt$
- Earth cools at about 50 to 100 K per Gyr
- Smaller bodies cool faster: lower thermal inertia, thinner boundary layer

All terrestrial planets are in net secular cooling; convection slows Earth's cooling but does not stop it.

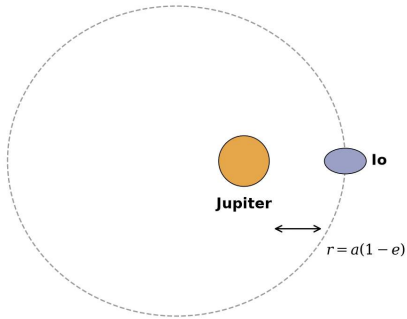
Tidal heating: eccentric orbits and the dissipation rate

$$\dot{E}_{\text{tidal}} = \frac{21}{2} \frac{k_2}{Q} \frac{GM_p^2 R^5 n e^2}{a^6}$$

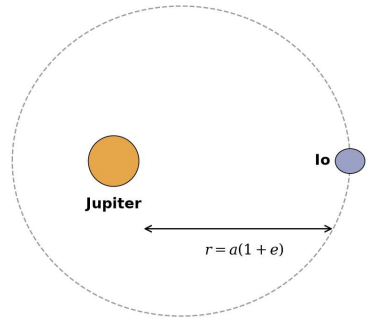
- k_2 : Love number (about 0.3 for rock); Q : tidal quality factor, lower means more dissipative; $n = \sqrt{GM_p/a^3}$: mean motion
- A circular, tidally locked orbit has a static bulge and no heating; eccentricity flexes the body every orbit
- At fixed e the heating scales as $a^{-15/2}$: dramatic for close-in moons

Cyclic flexing dissipates orbital energy as heat: $\dot{E}_{\text{tidal}} \propto e^2/Q$

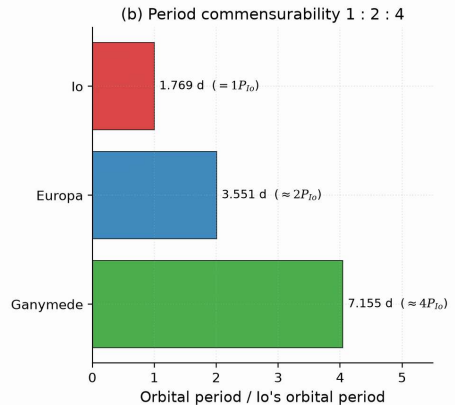
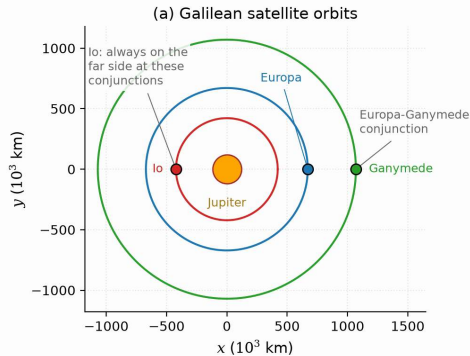
Periapsis: large tidal bulge



Apoapsis: small tidal bulge

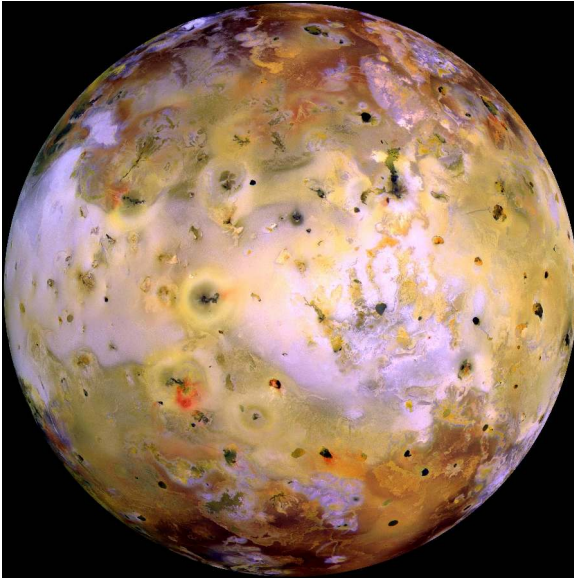


Tidal flexing on an eccentric orbit: the bulge is largest at periapsis and smallest at apoapsis; friction turns the cyclic work into heat. Course schematic.



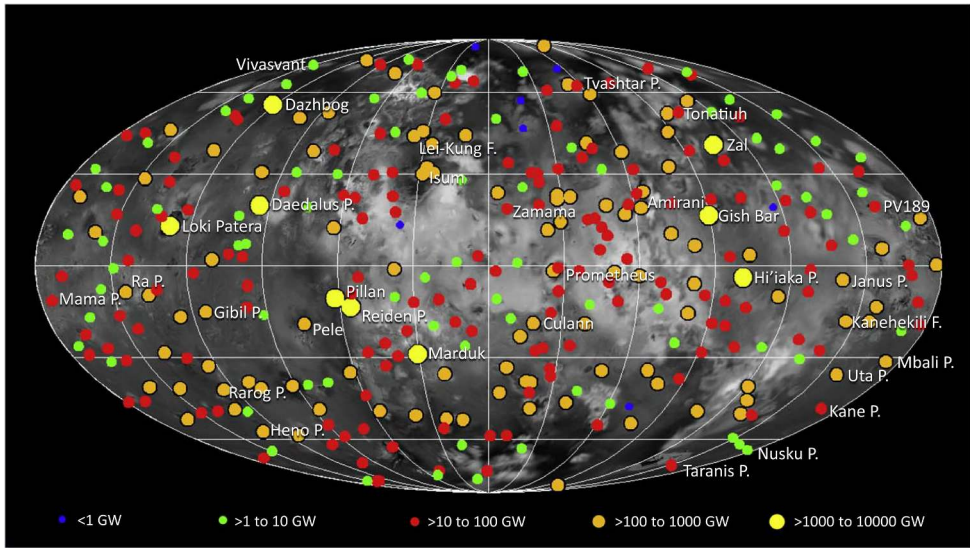
The Laplace resonance: (a) the three orbits at a Europa-Ganymede conjunction, (b) periods in the ratio 1:2:4. Course figure.

The 4:2:1 Io-Europa-Ganymede resonance (Laplace, 1805) pumps Io's eccentricity to about 0.004; alone, tides would circularise the orbit in about 10 Myr and the heating would stop.

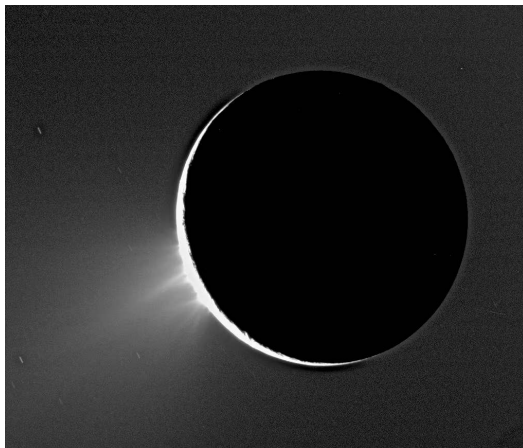


Io in Galileo colour: lava flows and sulfur deposits, no impact craters. Credit: NASA/JPL/University of Arizona, public domain.

About 10^{14} W of tidal heat, twice Earth's total, predicted by Peale, Cassen & Reynolds (1979) days before Voyager 1 saw the plumes; 343 active volcanic centres.



Io's 343 catalogued hot spots by power; Loki Patera alone emits 6000 to 12 000 GW, all together 58 ± 1 TW. Reproduced from Davies et al. (2024), Fig. 1.



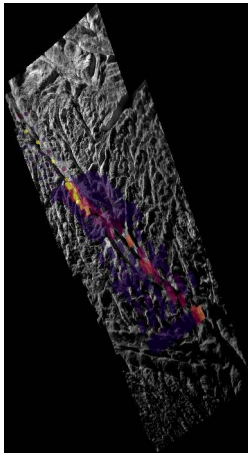
Water-ice jets from the “tiger stripe” fractures at the south pole of Enceladus. Credit: NASA/JPL/SSI (Cassini).

A 252 km moon with a global ocean under 15 to 25 km of ice and ~16 GW of tidal power: liquid water far outside the habitable zone.

Where in Enceladus is tidal heat dissipated?

- **Rocky core:** flexing of a porous silicate core; matches the hydrothermal silica (about 90°C)
- **Ice shell:** viscous dissipation at the warm shell base, 10 to 20 GW, focused at the south pole
- Likely both, with the ice shell dominating the plume heat flux

Source: Čadež et al. (2016); Hemingway & Mittal (2019)



Cassini CIRS thermal scan along one tiger-stripe fracture, draped on the visible mosaic.

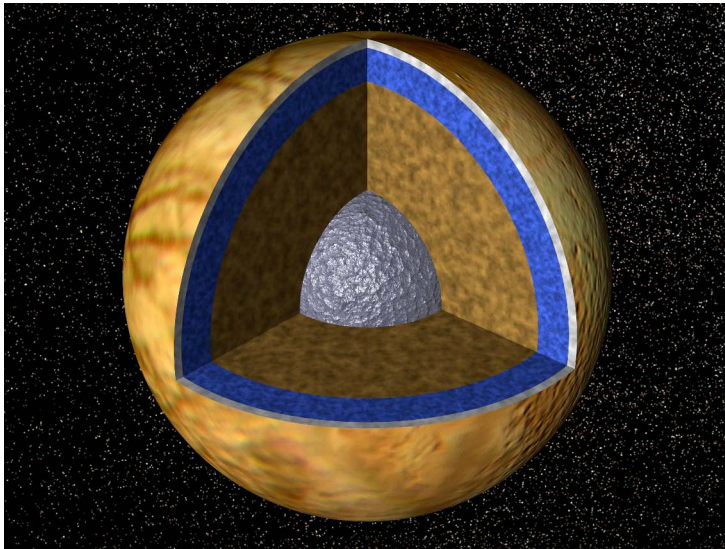
Credit: NASA/JPL/SSI.

15.8 ± 3.1 GW leave the south polar terrain through four fractures, not the plains, far beyond any radiogenic budget for a 252 km moon.

Ocean worlds: Enceladus, Europa and Titan

- **Enceladus** plume samples: salts, silica from a hot ocean floor, H_2 from serpentinisation, organics, phosphate (2023)
- **Europa**: ocean about 100 km deep under 15 to 25 km of ice; Europa Clipper (launched 2024)
- **Titan**: water-ammonia ocean from its rotational state; Dragonfly (about 2028)

Liquid water, chemical energy and organics: all the ingredients for chemoautotrophic life.

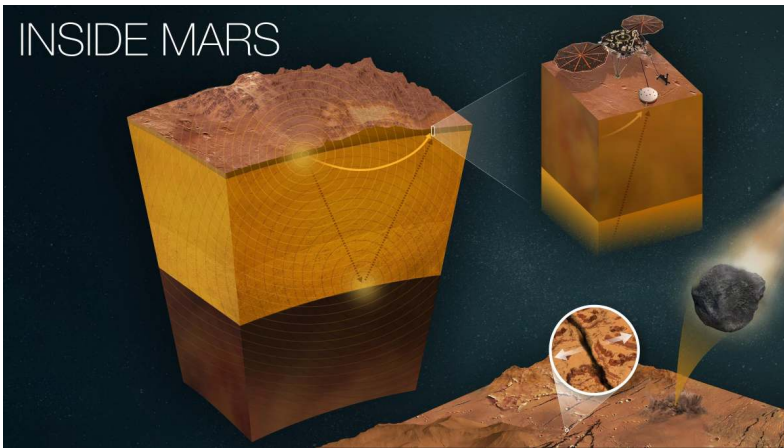


Europa's interior: iron-nickel core, silicate mantle, a global ocean about 100 km deep, and an ice shell of 15 to 25 km. Credit: NASA/JPL.

A grayscale image of the planet Io, showing its heavily cratered surface. A bright, elongated plume of material is visible on the left side of the planet, extending into space. The plume has a distinct, bright, and somewhat irregular shape, contrasting with the darker, cratered surface of the planet.

6. Recent advances & outlook

INSIDE MARS



InSight: seismic waves from marsquakes fix the crust, mantle and core of Mars; the 2023 reanalyses add a molten silicate layer above the core. Credit: NASA/JPL-Caltech.

Mars' thermal history sets its fate: dynamo shutdown at about 4.1 Ga, Tharsis volcanism, 15 to 25 mW m⁻² today, and an atmosphere lost with the field.

The 2030s at the ocean worlds and Io

- **Europa Clipper** (at Jupiter 2030): radar for the shell thickness, magnetometer for the ocean; shell or core dissipation?
- **JUICE** (Ganymede orbit 2034): ice-shell and ocean thickness, tidal heating from the Love number k_2
- **Juno at Io** (flybys 2023–2024): hotter poles than simple tidal models predict; gravity consistent with a global magma ocean

Source: Howell & Pappalardo (2020); Grasset et al. (2013); Park et al. (2024)

Summary

1. Four heat sources; Earth loses 47 TW, 40% radiogenic and 60% primordial
2. Conduction ($\tau \sim L^2/\kappa$) is too slow for planets: they convect, $Ra \gg Ra_c$, Earth as a mobile lid, the others as stagnant lids
3. Tidal heating powers Io and keeps the oceans of Enceladus, Europa and Titan liquid

Looking ahead to Lecture 4

This lecture followed the **heat**; Lecture 4 follows the **chemistry** it enables:

- How a molten planet separates into a metallic core and a silicate mantle
- What the Hf–W chronometer says about how *fast* core formation happened
- How a convecting metallic core generates the magnetic field that shields an atmosphere

Differentiation needs melting, and the dynamo needs a cooling core.

Questions?

Next: Lecture 4, Chemical differentiation & magnetospheres

Thu 17 Sep, 11:00–13:00, room 5419.0161 (Collegezaal)

Next tutorial: Fri 18 Sep, 13:00–15:00, room 5419.0161 (Collegezaal)

<https://ips.formingworlds.space>