

Lecture 8: Planetary Interiors Structure, Composition, & Dynamics

Planetary Systems

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Learning objectives

By the end of this lecture, you will be able to:

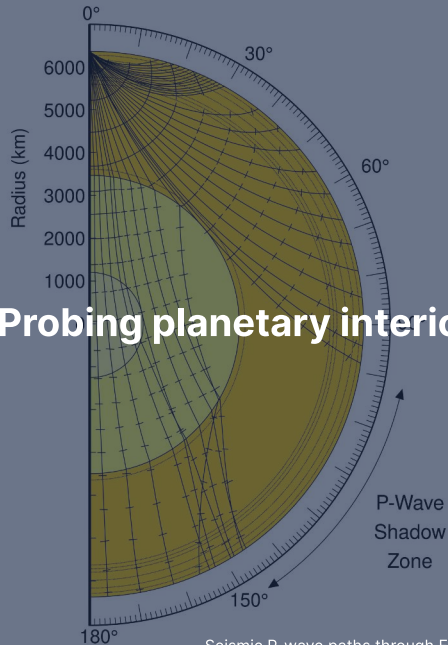
- Explain how seismology, gravity fields and the moment of inertia probe interiors
- Derive C/MR^2 for uniform and two-layer planets and apply hydrostatic equilibrium with an equation of state
- Compare internal structures across the solar system, including the InSight results for Mars

Recap: Lecture 7

- Crater scaling laws and surface chronology from crater counting
- Volcanism, tectonics, and erosion styles across the solar system
- Remote sensing and cryovolcanism on outer icy worlds

Today: What lies *beneath* the surface? How do we know, and what does it tell us?

1. Probing planetary interiors

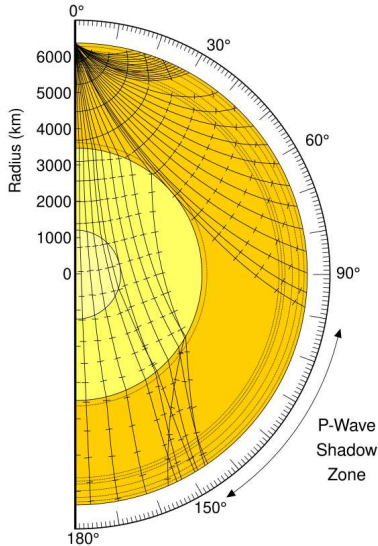


Seismic P-wave paths through Earth. Credit: Wikimedia Commons, public domain

Three ways to see inside a planet

1. **Seismology:** wave speeds trace density and elastic moduli
2. **Gravity fields:** spacecraft tracking measures the mass distribution (J_2)
3. **Moment of inertia:** degree of central mass concentration (C/MR^2)

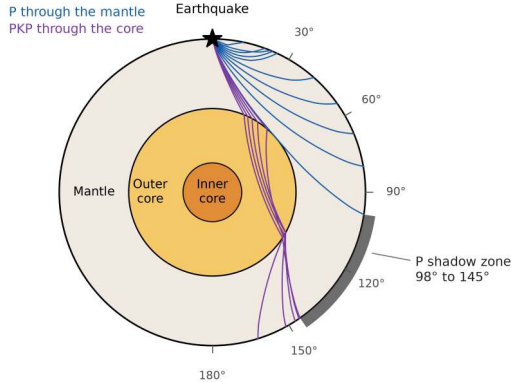
For Earth all three apply; for most other worlds, gravity and C/MR^2 are the only probes.



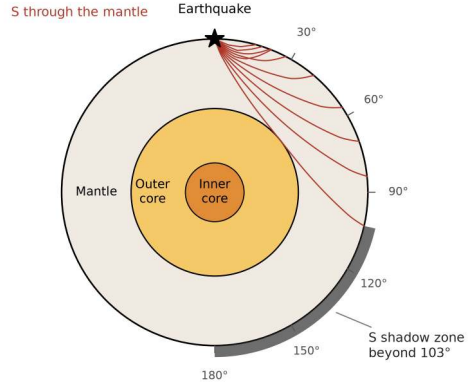
P-wave paths through Earth's interior; no direct P arrival reaches the surface in the shadow zone. Credit: Wikimedia Commons, public domain.

P-waves ($v_p = \sqrt{(K + \frac{4}{3}\mu)/\rho}$) travel through solids and liquids; S-waves ($v_s = \sqrt{\mu/\rho}$) need $\mu > 0$, so they **cannot propagate** through a liquid.

(a) P waves: refracted into the core



(b) S waves: stopped at the liquid core



Seismic rays traced through PREM, the Preliminary Reference Earth Model.
Course original, from the PREM tabulation of Dziewonski & Anderson (1981).

No S-waves beyond 103° proves the outer core is liquid; P-wave refraction opens a shadow zone, 98° to 145° in this ray tracing (textbook range 103° to 143°).

Gravity field measurements

- Rotational flattening produces a quadrupole potential term J_2
- Spacecraft radio tracking detects minute Doppler velocity shifts
- Juno and GRAIL constrain the internal mass distribution to high precision

The moment of inertia factor: C/MR^2

The moment of inertia factor measures central mass concentration:

Darwin-Radau relation:

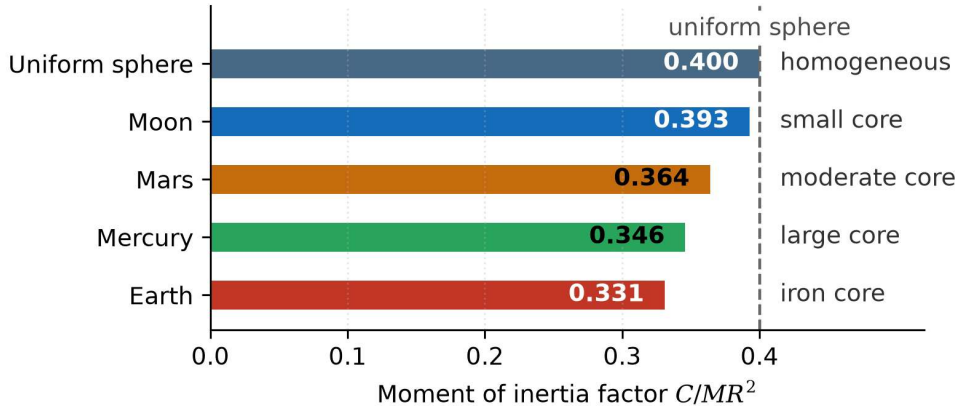
$$\frac{C}{MR^2} = \frac{2}{3} \left[1 - \frac{2}{5} \sqrt{\frac{4 - k_f}{1 + k_f}} \right]$$

- Hydrostatic bodies: from the flattening, $k_f = 3J_2/m$ with $m = \omega^2 R^3 / GM$
- Moon, Mars, Mercury: from J_2 and the precession or libration rate
- Lower value $\Rightarrow$ denser core

Body	C/MR^2	State
Uniform sphere	0.400	Homogeneous
Moon	0.393	Small core
Mars	0.364	Moderate core
Mercury	0.346	Large core
Earth	0.331	Iron core

Source: de Pater & Lissauer (2010)

The moment of inertia factor sorts bodies by central concentration

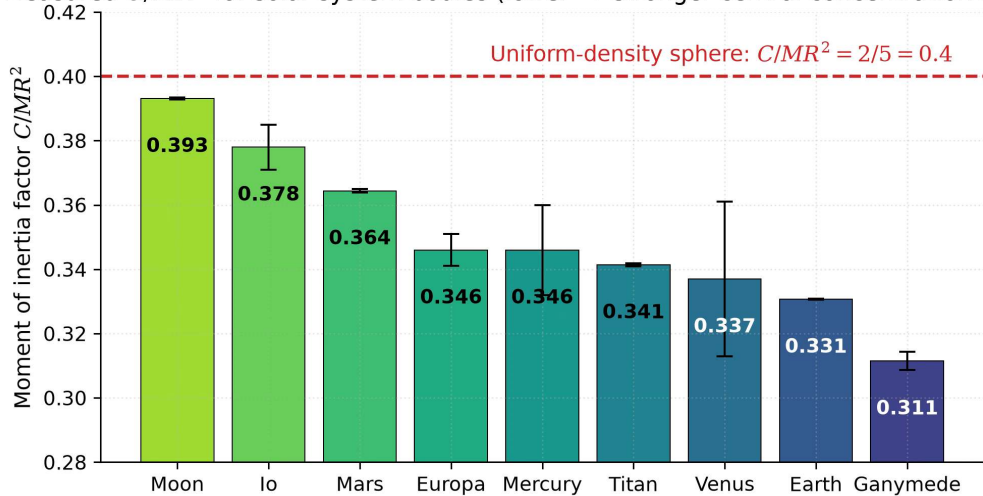


lower C/MR^2 means more mass concentrated toward the centre

The moment of inertia factor C/MR^2 from the uniform sphere (0.400) down to Earth (0.331), in the order of central mass concentration. Values after de Pater & Lissauer (2010).

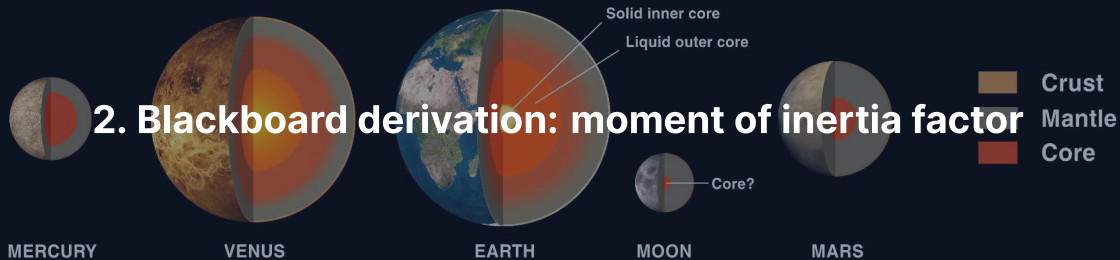
Course figure.

Measured C/MR^2 for solar-system bodies (lower $\Rightarrow$ stronger central concentration of mass)

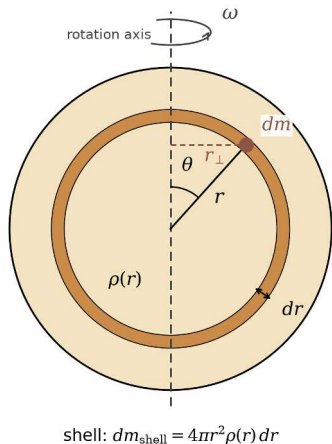


Moment of inertia factors for nine solar-system bodies, all below the uniform-sphere value of 0.400. Course original; lunar value from Williams et al. (2014).

2. Blackboard derivation: moment of inertia factor



Deriving C/MR^2



Goal: Derive the moment of inertia factor for (a) a uniform sphere and (b) a two-layer differentiated body.

Starting point: for a spherically symmetric body,

$$C = \frac{8\pi}{3} \int_0^R \rho(r) r^4 dr$$

A mass element at colatitude θ contributes $r_{\perp}^2 dm = r^2 \sin^2 \theta dm$; over a thin shell, $\sin^2 \theta \rightarrow 2/3$.

Credit: Course original

Summary of derived results

Uniform sphere: $\rho = \text{const}$

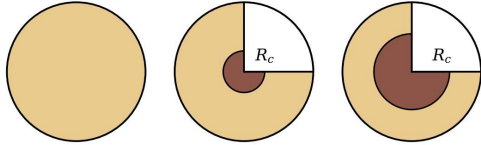
$$C = \frac{8\pi}{15}\rho R^5, \quad M = \frac{4}{3}\pi\rho R^3 \quad \Rightarrow \quad \frac{C}{MR^2} = \frac{2}{5} = 0.400$$

Two-layer body: core (ρ_c, R_c) and mantle (ρ_m, R), with $x \equiv R_c/R$ and $f \equiv \rho_c/\rho_m$:

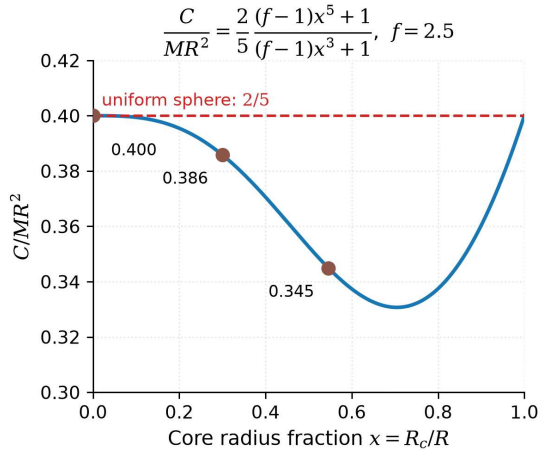
$$\frac{C}{MR^2} = \frac{2}{5} \cdot \frac{(f-1)x^5 + 1}{(f-1)x^3 + 1}$$

Since $x^5 < x^3$ for $0 < x < 1$, any dense core ($f > 1$) pulls $C/MR^2 < 0.400$.

$$C/MR^2 = 0.400 \quad C/MR^2 = 0.386 \quad C/MR^2 = 0.345$$

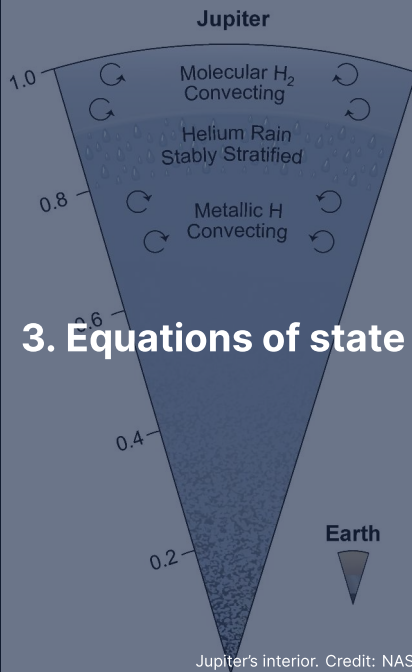


Uniform sphere Small dense core Large dense core
(Earth-like)
Same outer radius, density contrast $\rho_c/\rho_m = 2.5$



Two-layer body, $\rho_c/\rho_m = 2.5$: C/MR^2 against the core radius fraction $x = R_c/R$. Course original.

C weights mass by r^2 , so mass moved inward lowers C but not M : Earth ($x = 0.546$) gives $C/MR^2 \approx 0.345$, and the gap to the measured 0.331 is self-compression.



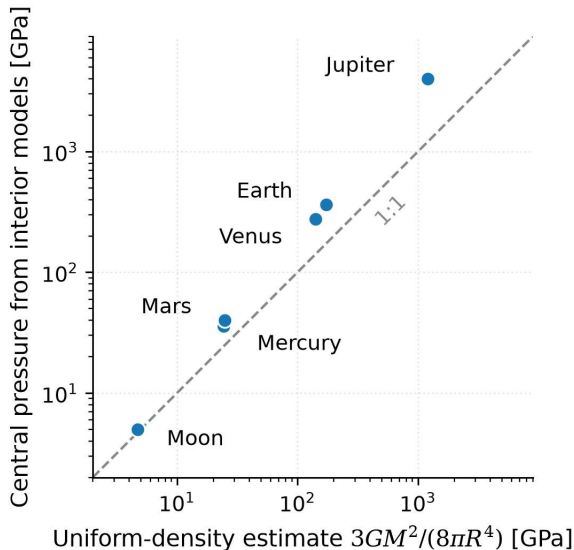
Hydrostatic equilibrium in spherical symmetry

Pressure at each depth supports the weight of the overlying material:

$$\frac{dP}{dr} = -\rho(r) g(r) = -\frac{G m(r) \rho(r)}{r^2}$$

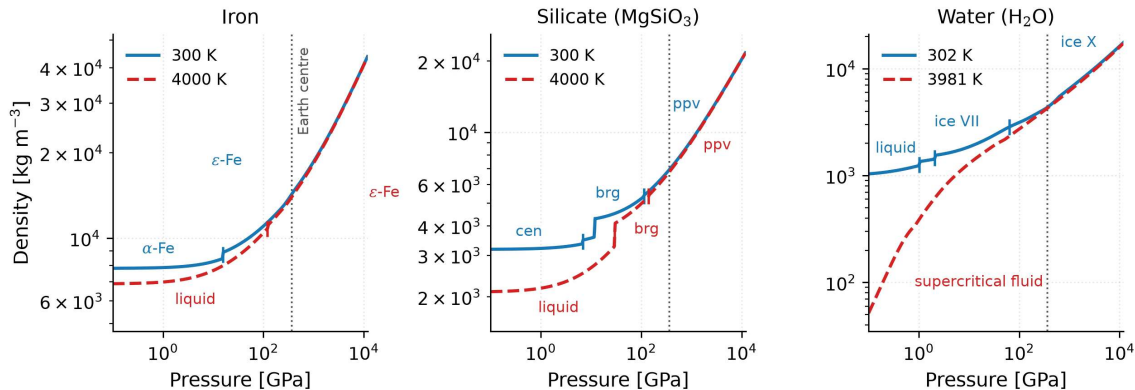
where $m(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$ is the mass enclosed within radius r .

With an equation of state $P = P(\rho, T)$, this integrates inward to the pressure-density profile.



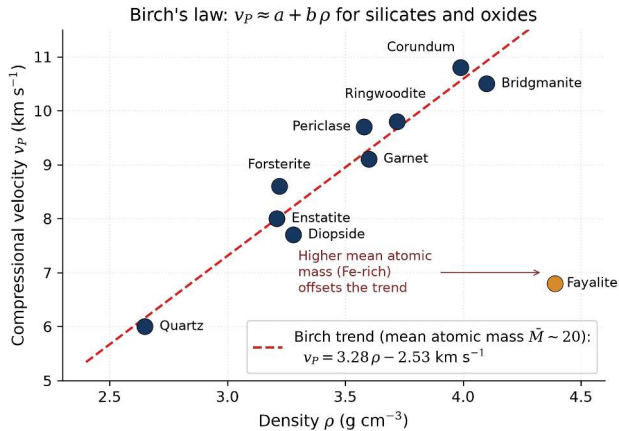
Central pressure from interior models against the uniform-density estimate for six solar-system bodies. Course original.

Uniform density gives $P_c \approx 3GM^2/(8\pi R^4)$, a lower bound: about 170 GPa against the actual 360 GPa for Earth, 25 against 40 GPa for Mars; real planets sit above the line because density increases inward.



Equations of state for iron, silicate and water on a cold (~ 300 K) and a hot (~ 4000 K) isotherm; the density jumps mark phase transitions.

Course original, from the PALEOS tables of Attia et al. (2026).



Birch's law: v_P against density for mantle minerals, trend after Birch (1952). Course original.

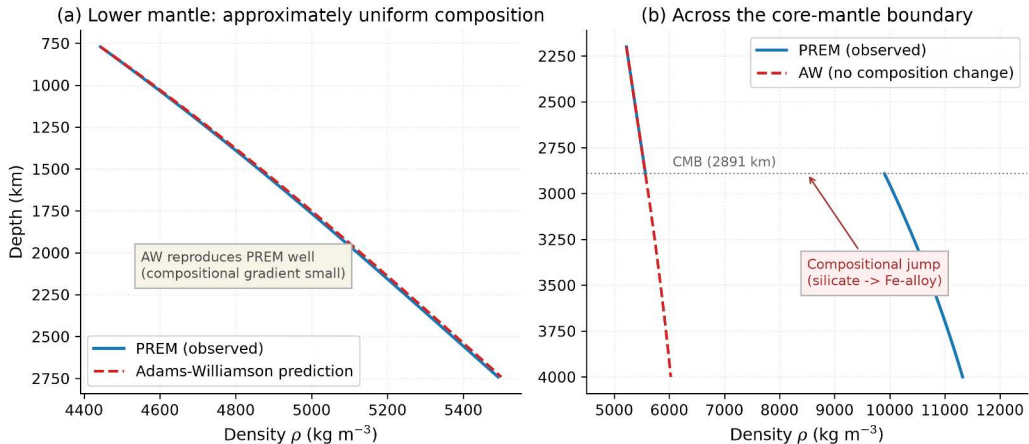
$v_P = a(\bar{M}) + b\rho$: at a given mean atomic weight $\bar{M}$, compressional velocity scales linearly with density, so seismic speeds map to density. Iron-rich minerals plot below the trend, so the law needs a known composition.

Adams-Williamson equation

Seismic wave speeds integrate to a density profile under adiabatic self-compression:

$$\frac{d\rho}{dr} = -\frac{\rho(r)g(r)}{\phi(r)}, \quad \phi = v_P^2 - \frac{4}{3}v_S^2$$

- Seismic parameter $\phi \equiv K_S/\rho = v_P^2 - \frac{4}{3}v_S^2$
- Assumes hydrostatic equilibrium, an adiabatic gradient and uniform composition
- Discontinuities mark phase changes or compositional boundaries (e.g., the core-mantle boundary)



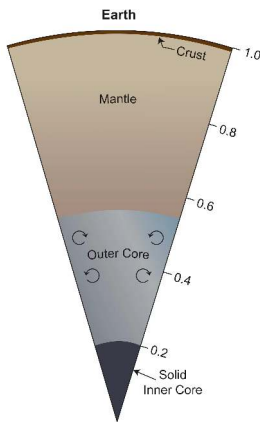
The Adams-Williamson relation against PREM: adiabatic self-compression matches the lower mantle, and the density jump at the core-mantle boundary exposes a change in composition.

Course original, from the PREM tabulation of Dziewonski & Anderson (1981).

4. Earth's interior

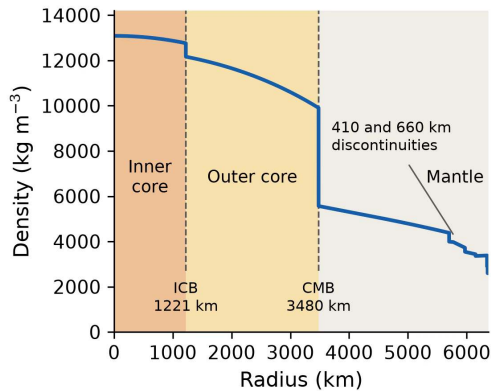


Earth's interior. Credit: NASA/JPL-Caltech/SwRI/John E. Connerney (PIA25063)



Earth's interior in cross-section. Credit: NASA/JPL-Caltech.

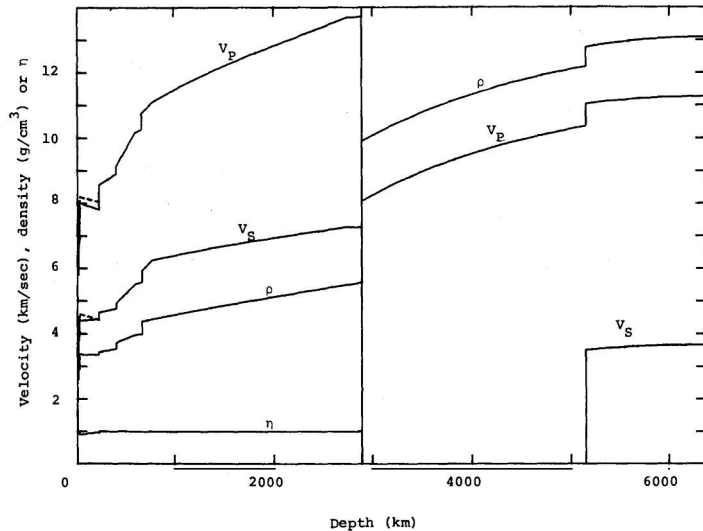
Crust (0–70 km), **mantle** (Moho to 2891 km), liquid **outer core** (2891–5150 km) and solid **inner core** (5150–6371 km, where the melting point at 330–360 GPa exceeds 5000–6000 K); mean density 5515 kg m^{-3} , $C/MR^2 = 0.3307$, central pressure about 360 GPa.



PREM density against radius.

Course original, from the PREM tabulation of Dziewonski & Anderson (1981).

Density rises from $\sim 3300 \text{ kg m}^{-3}$ in the upper mantle to ~ 5570 at its base, jumps to $9900\text{--}12,200$ in the outer core and reaches $\sim 13,000$ in the inner core. The 410 and 660 km steps are phase transitions; the core-mantle boundary is silicate to iron.

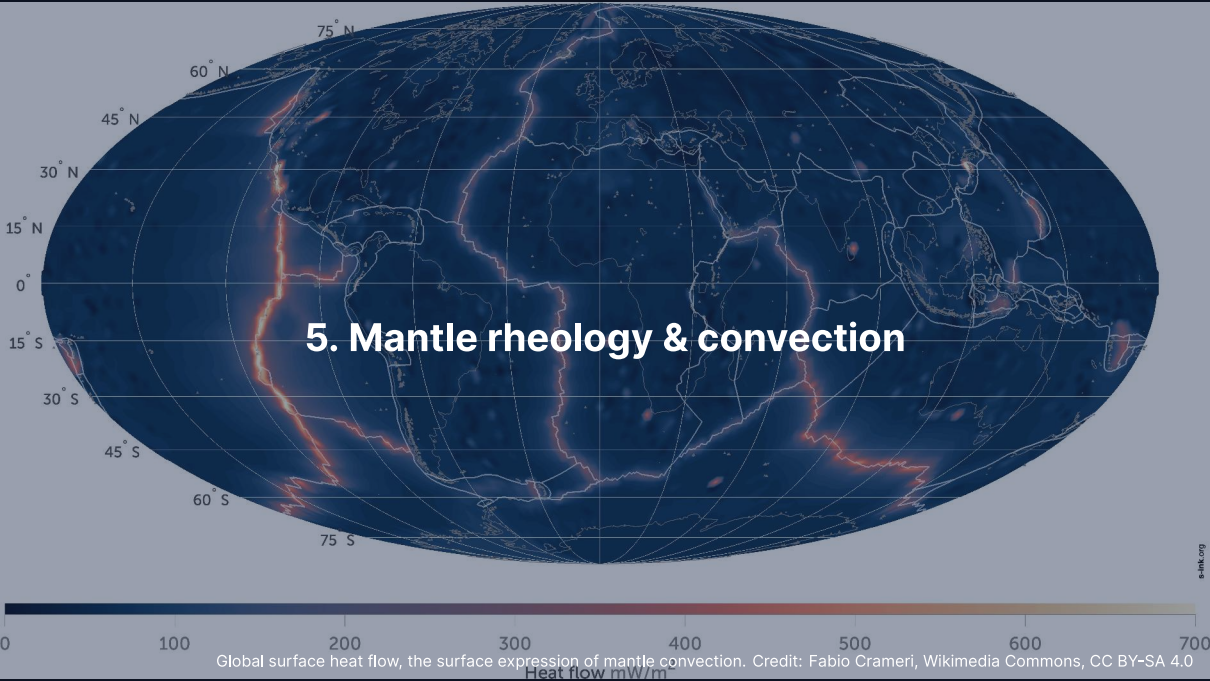


PREM radial profiles of wave speeds, density and anisotropy; v_s vanishes in the liquid outer core and resumes in the solid inner core. Credit: Dziewonski & Anderson (1981), Fig. 8.

Crust, mantle, and core

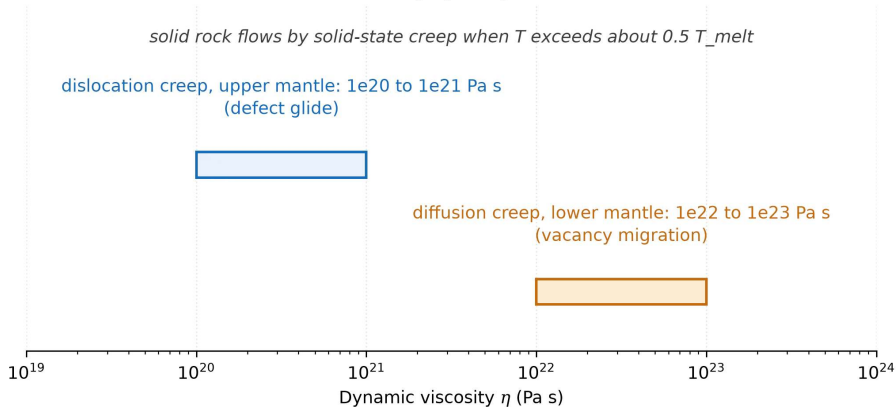
Layer	Depth (km)	ρ (kg m ⁻³)	Composition
Crust	0–70	2700–3000	Silicates (Moho base)
Upper mantle	70–410	3300–3700	Olivine, pyroxene
Transition zone	410–660	3700–4400	Wadsleyite, ringwoodite
Lower mantle	660–2891	4400–5500	Bridgmanite
Outer core	2891–5150	9900–12,200	Liquid Fe-Ni
Inner core	5150–6371	~13,000	Solid Fe-Ni

Convection in the liquid outer core powers the geodynamo, fed by inner-core growth: crystallising iron releases latent heat and light elements.



Global surface heat flow, the surface expression of mantle convection. Credit: Fabio Cramer, Wikimedia Commons, CC BY-SA 4.0

Mantle viscosity by creep mechanism



Mantle viscosity by creep mechanism. Course schematic.

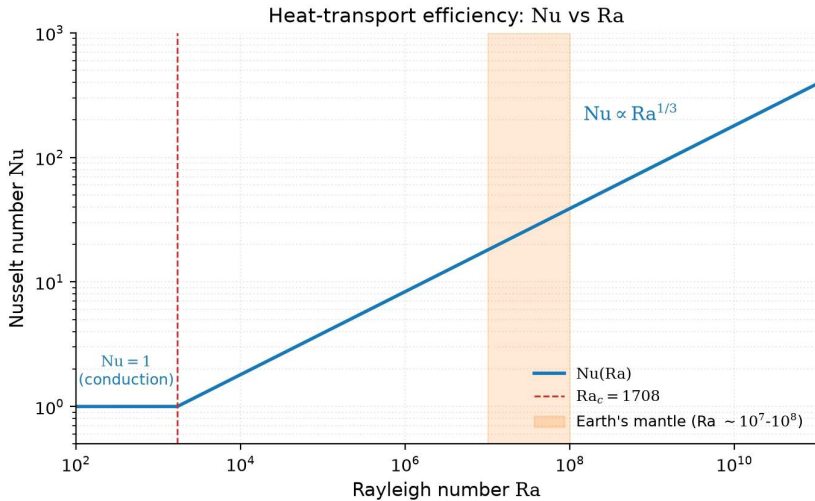
Solid rock flows once $T > 0.5 T_{\text{melt}}$: **dislocation creep** (defect glide) in the upper mantle at 10^{20} to 10^{21} Pa s, **diffusion creep** (vacancy migration) in the lower mantle at 10^{22} to 10^{23} Pa s.

The Rayleigh number

Thermal buoyancy against viscous resistance decides whether a mantle convects:

$$Ra = \frac{\alpha \rho g \Delta T d^3}{\kappa \eta}$$

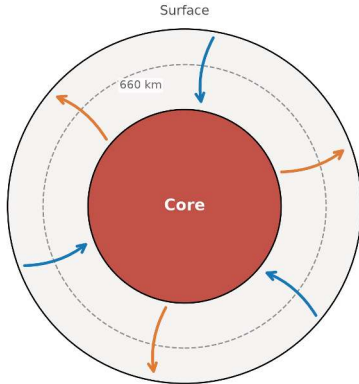
- Ra is the ratio of buoyancy driving forces to viscous dissipation
- Critical threshold for the onset of convection: $Ra_c \sim 10^3$
- Earth's mantle: $Ra \sim 10^7 - 10^8 \gg Ra_c$, vigorous convection



$Nu = 1$ below Ra_c , then $Nu \propto Ra^{1/3}$: Earth's mantle at $\eta = 10^{21}$ to 10^{22} Pa s (shaded) sits at $Ra \approx 10^7$ to 10^8 , four to five orders of magnitude above Ra_c , and moves heat 18 to 39 times faster than conduction. Course figure, first shown in Lecture 3.

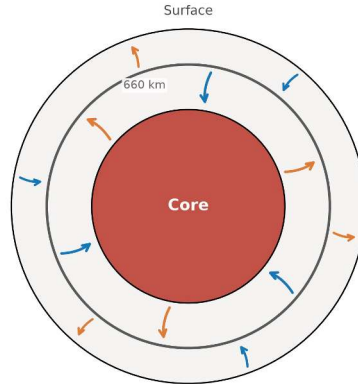
Mantle convection regimes

(a) Whole-mantle convection



Single circulation
from CMB to surface

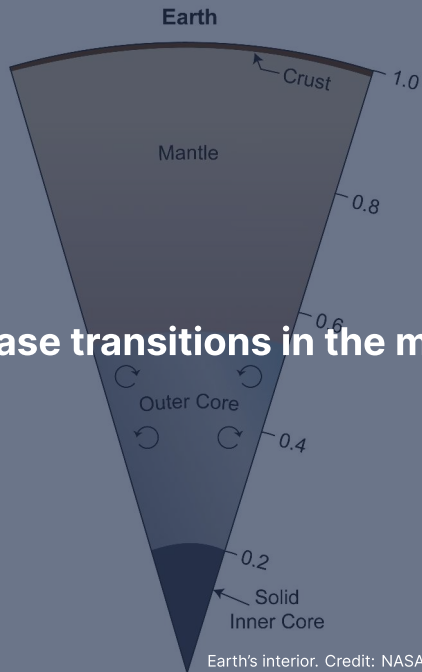
(b) Layered convection



Upper- and lower-mantle
cells separated by 660 km

End-member regimes of mantle convection; Earth convects mainly as a whole mantle, with the 660 km discontinuity a partial, regionally variable barrier. Schematic; boundary radius exaggerated.

6. Phase transitions in the mantle

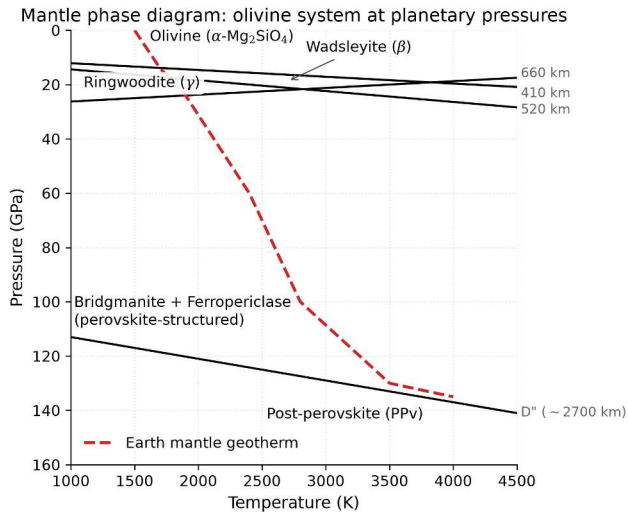


Earth's interior. Credit: NASA/JPL-Caltech/SwRI/John E. Connerney (PIA25063)

Mineral phase transitions with depth

Depth (km)	Transition	$\Delta\rho$
410	Olivine $\rightarrow$ wadsleyite	~5%
520	Wadsleyite $\rightarrow$ ringwoodite	~2%
660	Ringwoodite $\rightarrow$ bridgmanite + ferropericlase	~8%
~2700	Bridgmanite $\rightarrow$ post-perovskite	~1–2%

- At 410 and 520 km olivine ($(\text{Mg}, \text{Fe})_2\text{SiO}_4$) turns into denser polymorphs of the same composition; at 660 km ringwoodite splits into bridgmanite and ferropericlase
- Driven by pressure: atoms rearrange into denser packing
- These transitions produce the primary seismic discontinuities in PREM



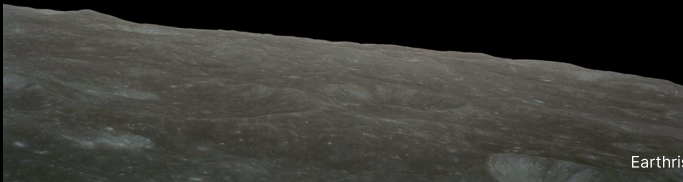
Phase boundaries of the mantle against an approximate geotherm; seismic discontinuities occur where the geotherm crosses a phase boundary. After Turcotte & Schubert (2002) and Credit: Murakami et al. (2004).

The 660 km discontinuity

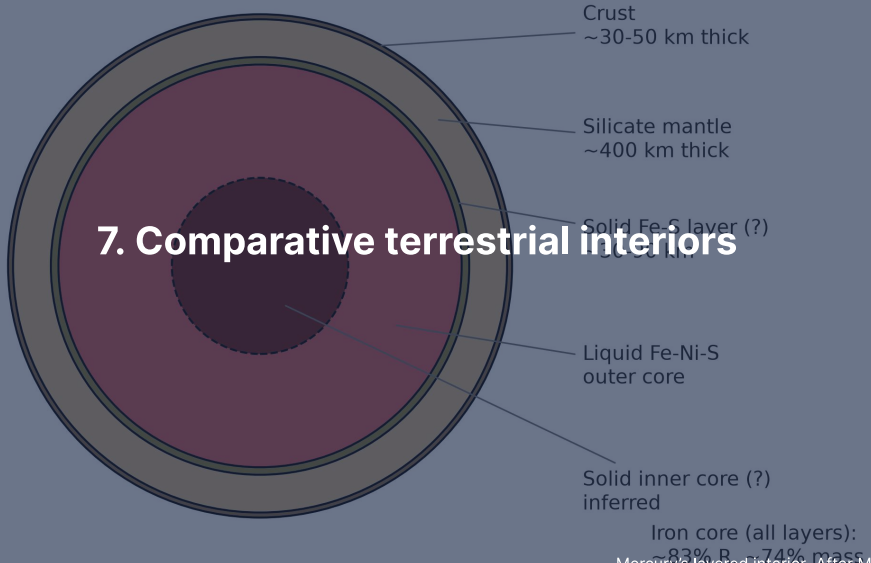
- Ringwoodite decomposes into bridgmanite plus ferropericlasite with an ~8% density jump
- Negative Clapeyron slope ($dP/dT < 0$): in a cold slab the boundary sits deeper, so buoyant ringwoodite persists below 660 km
- The upward buoyancy is a partial barrier to whole-mantle convection

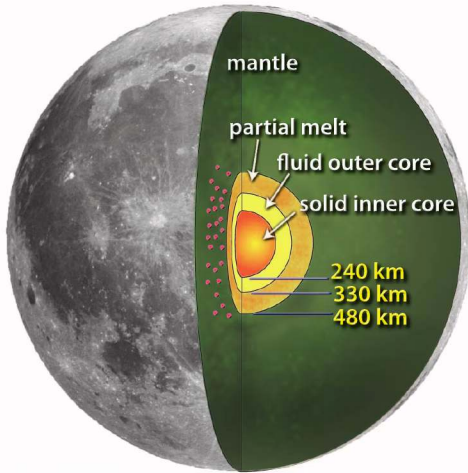
The endothermic ringwoodite breakdown at 660 km resists slab penetration.

Break



Mercury's interior structure (post-MESSENGER model)

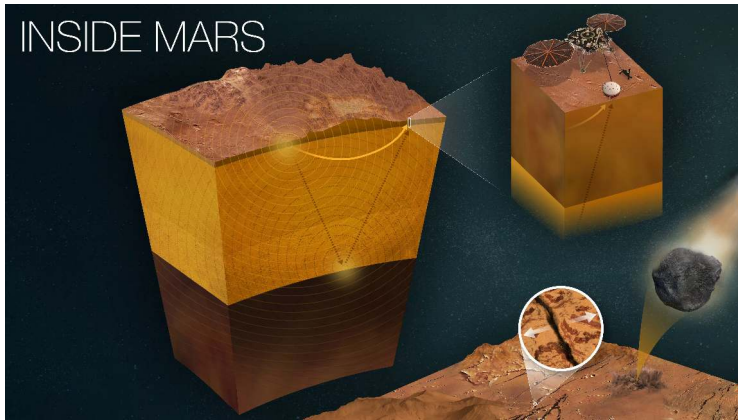




The lunar interior from reanalysed Apollo seismograms: a small metallic core with a liquid outer and a solid inner part. Credit: Weber et al. (2011), Fig. 1A.

$C/MR^2 = 0.393$ and
 $\bar{\rho} = 3346 \text{ kg m}^{-3}$, close to
uncompressed mantle rock: the
core is only 200–380 km in
radius ($R_c/R \sim 0.1\text{--}0.2$, < 2% of
the mass), as expected for a
Moon formed by a giant impact
from Earth's mantle.

INSIDE MARS

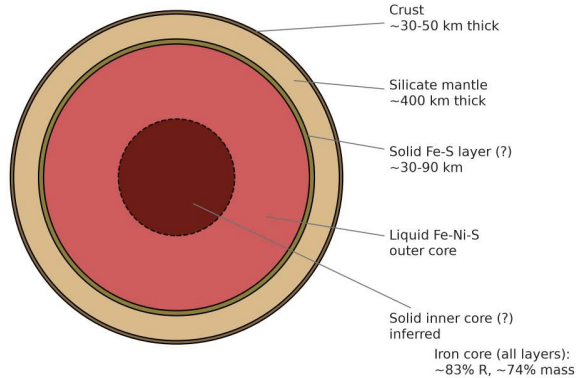


Mars's interior from InSight: 24–72 km crust, mantle to ~1560 km depth, core.

Credit: NASA/JPL-Caltech.

The seismic boundary at $R_c \sim 1830$ km was reinterpreted in 2023 as the top of a molten silicate layer above a smaller (~1650–1675 km) liquid iron core of about 6500 kg m^{-3} .

Mercury's interior structure (post-MESSENGER model)



Mercury's interior from MESSENGER gravity and libration. After Margot et al. (2018).

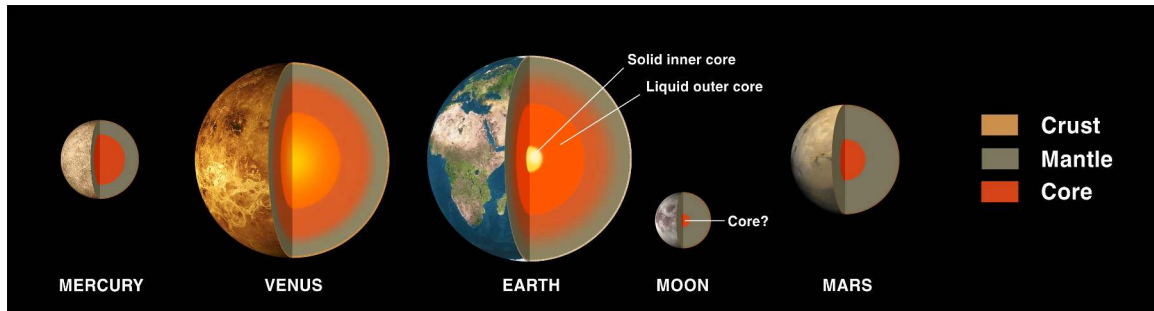
$C/MR^2 = 0.346$ and $\bar{\rho} = 5427 \text{ kg m}^{-3}$, the highest uncompressed density of the terrestrial planets: the metallic core reaches $R_c \sim 2000\text{--}2050 \text{ km}$ (~83% of the radius, ~74% of the mass), and the weak dynamo indicates a liquid outer core.

Comparing terrestrial body interiors

Body	$\bar{\rho}$ (kg m ⁻³)	C/MR^2	R_c/R	Dynamo?
Moon	3346	0.393	~0.1–0.2	No (ancient)
Mars	3934	0.364	~0.49–0.54	No (ancient)
Venus	5243	~0.33	~0.55	No
Earth	5515	0.331	0.55	Yes
Mercury	5427	0.346	~0.83	Weak

Mars: upper value from Stähler et al. (2021), lower from the 2023 reanalyses (Khan et al.; Samuel et al.); Mercury: Margot et al. (2018)

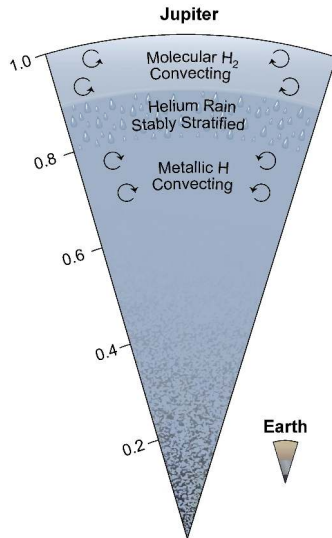
- Higher *uncompressed* density $\Rightarrow$ larger iron core fraction; self-compression lifts Earth's mean density above Mercury's
- Lower $C/MR^2 \Rightarrow$ stronger central mass concentration
- A dynamo requires a convecting liquid conductor



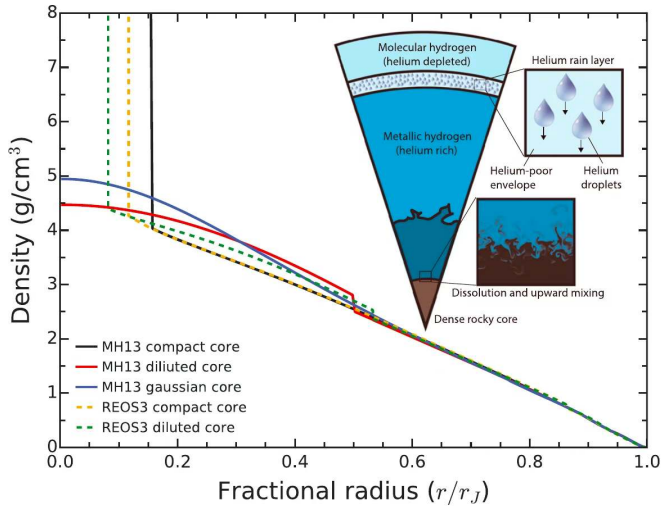
The terrestrial bodies in cross-section at relative scale: a silicate mantle over a metallic core, with core fractions from ~10–20% to ~83%.

Credit: NASA Solar System Exploration, public domain.

8. Giant planet interiors

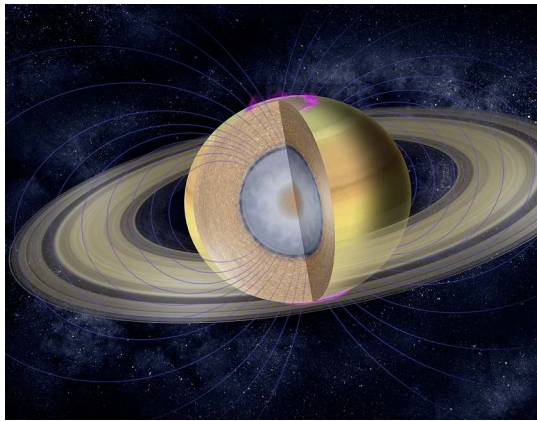


Jupiter's interior: molecular and metallic hydrogen above an extended, dilute core enriched in heavy elements. Credit: NASA/JPL-Caltech/SwRI/John E. Connerney (PIA25062).



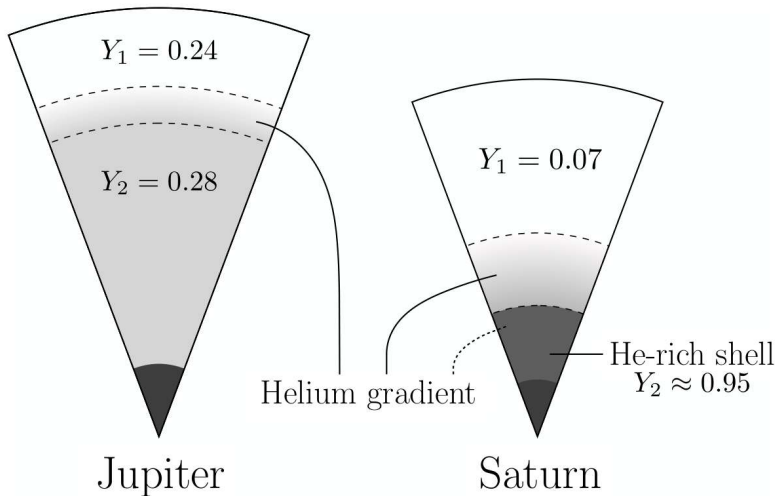
Density profiles of Jupiter interior models from Juno gravity: dilute-core solutions spread heavy elements out to half the radius, against classical compact cores.

Credit: Wahl et al. (2017), Fig. 1.



Artist's cutaway of Saturn's interior; ring seismology puts a diffuse core at $\sim 60\%$ of the radius (Mankovich & Fuller 2021). Credit: NASA/JPL-Caltech.

As Saturn cools, helium becomes immiscible in metallic hydrogen at $P \sim 2$ Mbar and rains toward the centre; the released gravitational energy, with Kelvin-Helmholtz contraction, explains why Saturn radiates $\sim 1.8\times$ the solar flux it absorbs.

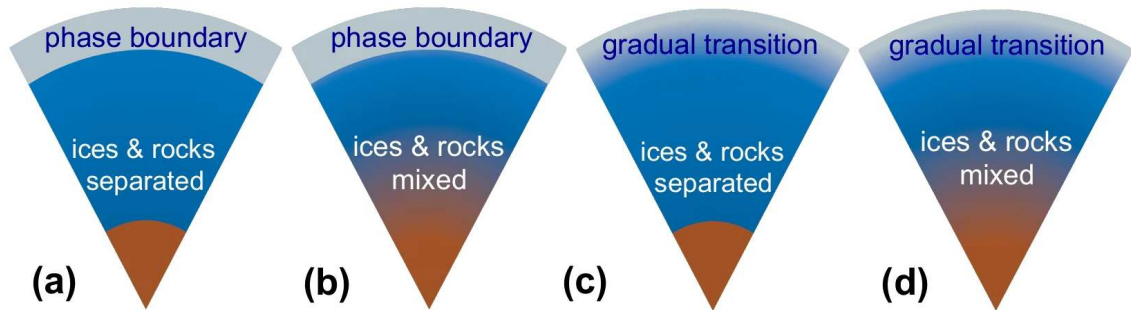


Helium phase separation in Jupiter and Saturn; sinking helium droplets release gravitational potential energy and explain Saturn's excess luminosity.

Credit: Mankovich & Fortney (2020), Fig. 3.

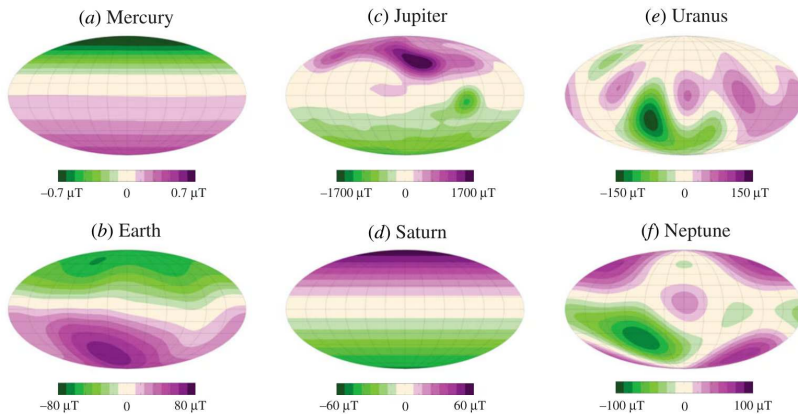


9. Ice giant interiors



Four candidate internal structures of an ice giant. Credit: Helled et al. (2020), Fig. 4.

Canonical layers: an H/He envelope (~20% of the mass), a fluid ice mantle (~60–70%: H_2O , NH_3 , CH_4 at 10–500 GPa, possibly superionic) and a rocky core; gravity alone cannot separate sharp layers from gradients.



Surface radial field of six planets in Mollweide projection.

Credit: Soderlund & Stanley (2020), Fig. 1.

Uranus and Neptune generate multipolar fields tilted by 59° and 47° and offset from the centre, from dynamos in thin outer fluid ice shells, unlike the axial dipoles of Earth, Jupiter and Saturn.

10. Icy moon interiors & ocean worlds



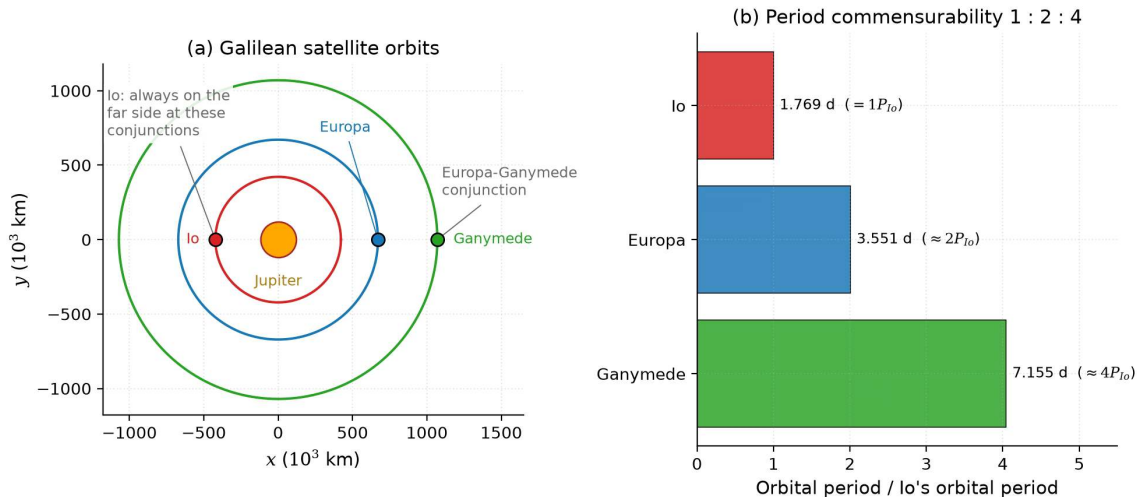
Ocean worlds: liquid water beneath ice

Tidal flexing and radiogenic decay sustain subsurface oceans beneath insulating ice shells:

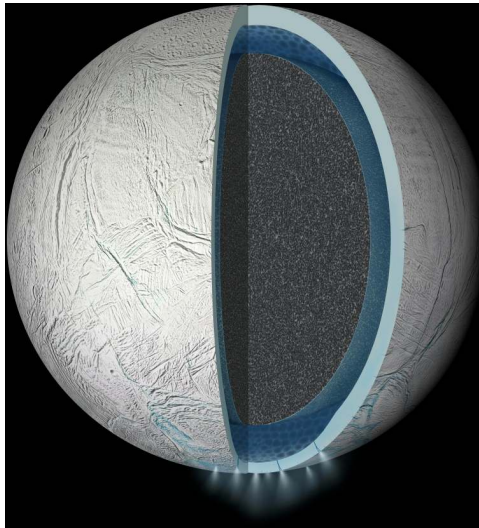
Moon	R (km)	ρ (kg m⁻³)	Ocean evidence
Europa	1561	3013	Magnetic induction, chaos terrain
Enceladus	252	1609	Plumes, libration
Titan	2575	1882	Tidal Love number k_2
Ganymede	2634	1936	Auroral oscillations



Europa's subsurface environment: an ice shell over a global ocean roughly 100 km deep in contact with a rocky seafloor. Credit: NASA/JPL-Caltech (PIA26438).

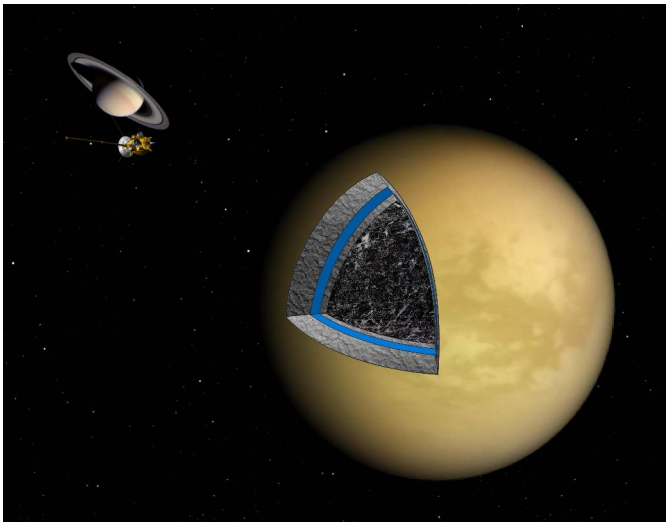


The Laplace resonance: (a) the three orbits at a Europa-Ganymede conjunction, (b) periods in the ratio 1:2:4. The 4:2:1 resonance forces the eccentricities whose tidal flexing keeps Europa's ocean liquid. Course figure, first shown in Lecture 3.



Enceladus in cutaway: a global ocean between the ice shell and a porous core feeds hydrothermal venting and south-polar plumes radiating ~16 GW.

Credit: NASA/JPL-Caltech (PIA19656).

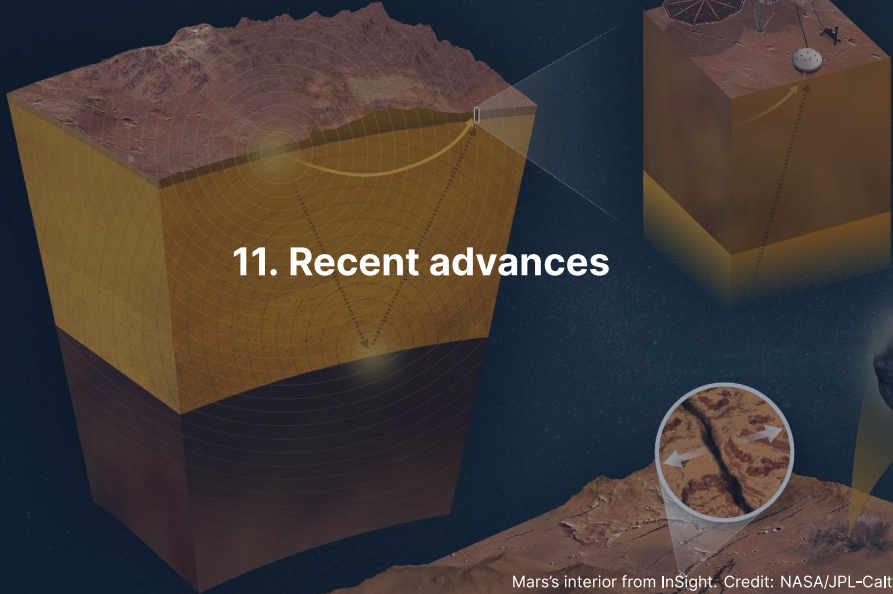


Titan's interior from Cassini gravity and radio science: a large tidal Love number k_2 indicates an ocean that decouples the outer ice shell from the high-pressure ice and rock below.

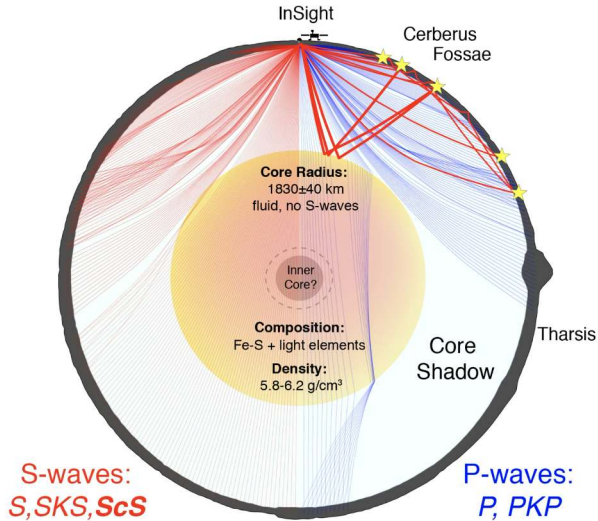
Credit: NASA/JPL.

INSIDE MARS

11. Recent advances

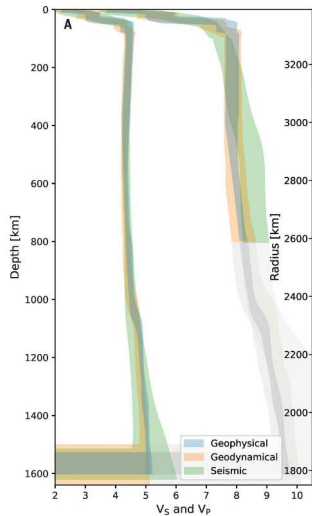


Mars's interior from InSight. Credit: NASA/JPL-Caltech (PIA25282)



InSight seismic ray paths across Mars delineate the mantle and the core shadow zone; the 2023 reanalyses put a molten silicate layer above a ~1650–1675 km core.

Credit: Stähler et al. (2021), Fig. 3.



Mars velocity profiles from InSight: the mantle v_s and v_p profiles end at the core boundary near 1560 km depth, below which no shear wave propagates, a liquid metallic core.

Credit: Stähler et al. (2021), Fig. 2A.

Summary

1. Seismology, gravity fields and C/MR^2 probe deep interiors: 0.400 for a uniform sphere, lower values mean differentiation
2. Hydrostatic equilibrium and equations of state set the radial profiles; mantle phase transitions at 410 and 660 km shape convection
3. Tidal heating maintains subsurface oceans across the icy worlds

Next: Lecture 9 turns to Earth and Venus: twins in size, contrasts in evolution.

Questions?

Next: Lecture 9, Rocky planets I: Earth & Venus

Mon 5 Oct, 15:00–17:00, room 5419.0161 (Collegezaal)

Next tutorial: Fri 2 Oct, 13:00–15:00, room 5419.0161 (Collegezaal)

<https://ips.formingworlds.space>

